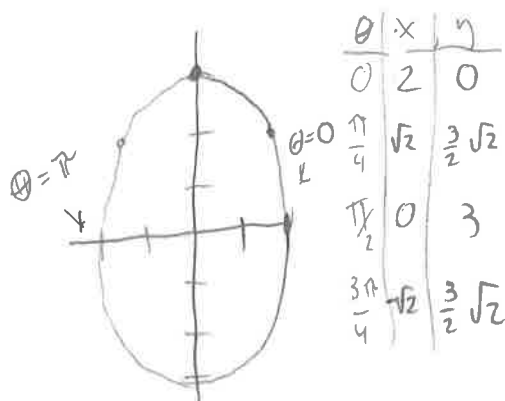


Show your work fully for all questions. Quiz has front and back sides.

Problem 1: An ellipse is given by $x = a \cos \theta$, $y = b \sin \theta$. For $a = 2$ and $b = 3$ sketch the ellipse and find the area it encloses.



$$\text{Area} = 2 \cdot \text{Area of top half}$$

$$= 2 \cdot \int_{\pi}^0 y(\theta) \frac{dx}{d\theta} d\theta$$

$$= 2 \cdot \int_{\pi}^0 b \sin(\theta) \cdot -a \sin(\theta) d\theta$$

$$= 2ab \int_0^{\pi} \sin^2 \theta d\theta$$

$$= 2ab \int_0^{\pi} \frac{1 - \cos(2\theta)}{2} d\theta = 2ab \left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4} \right) \Big|_0^{\pi}$$

$$= 2ab \frac{\pi}{2} + 0 = 6\pi$$

Problem 2: Find the length of the curve given by $x = t^3$, $y = 2t$ for $0 < t < 2$.

$$L = \int_{t_0}^{t_1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_0^2 \sqrt{(3t^2)^2 + (2)^2} dt$$

$$= \int_0^2 \sqrt{9t^4 + 4} dt$$

= Very hard to solve

Problem 3: Find the slope of the tangent line of $r = 3 - 2\sin\theta$ at $\theta = \pi/3$

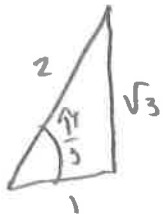
$$\text{Slope} = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$x(\theta) = r \cos \theta = (3 - 2\sin\theta) \cos \theta$$

$$\frac{dx}{d\theta} = (-2 \cos \theta) \cos \theta + (3 - 2\sin\theta) \cdot -\sin \theta = -3 \sin \theta + 2 \sin^2 \theta - 2 \cos^2 \theta$$

$$y(\theta) = r \sin \theta = (3 - 2\sin\theta) \sin \theta$$

$$\frac{dy}{d\theta} = (-2 \cos \theta) \sin \theta + (3 - 2\sin\theta) \cos \theta = 3 \cos \theta - 4 \cos \theta \sin \theta$$



$$\text{So } \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\text{Thus } \frac{dy}{d\theta} = 3 - 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2}, \quad \frac{dx}{d\theta} = -3 \frac{\sqrt{3}}{2} + 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 - 2 \cdot \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{2} - \frac{2\sqrt{3}}{2}$$

$$= -\frac{3\sqrt{3}}{2} + 2 \cdot \frac{3}{4} - \frac{2}{4}$$

$$= -\frac{3\sqrt{3}}{2} + \frac{2}{2}$$

$$\text{Finally } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{3}{2} - \frac{2\sqrt{3}}{2}}{\frac{2}{2} - \frac{3\sqrt{3}}{2}} = \boxed{\frac{3 - 2\sqrt{3}}{2 - 3\sqrt{3}}}$$