The conformal manifold in 2d $N = (2,2)$ SCFTs

Theory and Methods

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Outline

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- The assignment of meaning to the deformation by $\lambda \mathcal{O}(x)$ depends on a renormalization scheme. Different renormalization schemes correspond to reparametrizations of λ.

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- Free bosons on the torus come in a one-dimensional family parametrized by r.

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- Can define a Riemannian metric called the Zamolodchikov metric:
$$g_{\mu\nu} := \langle \mathcal{O}_\mu(1)\mathcal{O}_\nu(0)\rangle$$
- Theories with supersymmetry tend to have rich spaces of exactly marginals that preserve SUSY, enough to add more structure (complex, Kähler).

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- Roughly speaking, in localization, the path integral turns out to be independent of $\hbar$; we can therefore take the $\hbar \to 0$ (semi-classical) limit. This is why 1-loop calculations turn out exact.

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- Computing the S^2 partition function while preserving $SU(2 \mid 1)_A$ or $SU(2 \mid 1)_B$ yields the Kähler potential for either S_c or S_{tc}:

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$$Z^A_{S^2} = e^{-\mathcal{K}_{tc}}, Z^B_{S^2} = e^{-\mathcal{K}_C}.$$
- GLSMs can be coupled to S^2. The partition function computation localizes and is an RG-invariant. Thus, we can compute the Kähler potential of the superconformal manifold for superconformal NLSMs arising as low-energy limits of GLSMs.

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- The conformal manifold gives an analytic structure on the space of CFTs that allows us to probe theories near a given well-understood CFT.
- 2d $\mathcal{N} = (2,2)$ SCFTs are interesting because many arise as string compactifications on Calabi-Yau manifolds.

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- A CFT deformation is a shift in the $\langle\Sigma|$ such that the smoothness and sewing conditions remain satisfied.
- In the conformal manifold, a choice of infinitesimal deformation at each point gives a connection on the bundle of operators over the conformal manifold.

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- New formalism allows us to give precise meaning to renormalization schemes and higher-order corrections within context of CFT.
- Deformations arise as connections $\nabla_\mu := \partial_\mu + \Gamma_\mu$ on the conformal manifold.
- Define the finitely deformed surface states

$$\langle\Sigma|_{new} = \exp\left(\lambda^\mu \nabla_\mu\right) \langle\Sigma| \,.$$

Connections, Renormalization, and Higher-Order Corrections: 2

- Higher-order terms in perturbation theory are just higher-derivatives of the $\langle\Sigma|$. In particular, the second-order term is

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Connections, Renormalization, and Higher-Order Corrections: 2

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$$\frac{1}{2}\lambda^{\mu}\lambda^{\nu}\left(\partial_{\mu}\Gamma_{\nu}+\Gamma_{\mu}\Gamma_{\nu}\right)|P\rangle$$
- Don't have to worry about colliding disks that arise in point-splitting. If the coefficients of Γ_{μ} are known, it is completely straightforward to compute higher-order contributions in perturbation theory. This will be the case for the "regularization scheme" we consider below.

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  regulating the integral by continuing in the dimension of $\mathcal{O}_\mu$ and extending by Leibniz rule to the $\langle\Sigma|$.
- More explicitly, given a basis $\{\phi_i\}$ of operators with scaling dimension Δ_i define the connection:

$$\Gamma_{\mu j}{}^{i} = \begin{cases} H_{\mu j}{}^{i} \frac{2^{3-\Delta} r^{2-\Delta}}{2-\Delta} & \Delta_i \neq \Delta_j \\ 0 & \Delta_i = \Delta_j \end{cases} \, ,$$

  with $\Delta = 2 + \Delta_j - \Delta_i$ and $H_{\mu j}{}^{i}$ the 3-point function coefficients.

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- The deformation only preserves SUSY if deformation restricted to half of the $\mathcal{O}_\mu$. Get either $\mathcal{K}_c$ or $\mathcal{K}_{tc}$.

Outline

Localization: the General Theory 1

Theorem

Suppose a theory has a symmetry $\mathcal{Q}$. Correlators of $\mathcal{Q}$-closed operators ϕ_i are invariant under shifts by $\mathcal{Q}$-exact terms. In particular, the correlator of a $\mathcal{Q}$-exact operator with $\mathcal{Q}$-closed operators is 0.

Proof.

$$\delta\left\langle\prod_{\alpha}\phi_\alpha\right\rangle = \left\langle\{\mathcal{Q},\varphi\}\cdot\prod_{\alpha\neq\alpha'}\phi_\alpha\right\rangle$$

$$= \left\langle\left\{\mathcal{Q},\varphi\cdot\prod_{\alpha\neq\alpha'}\phi_\alpha\right\}\right\rangle - \sum_{\beta}\langle\varphi\cdot\{\mathcal{Q},\phi_\beta\}\cdot\prod_{\alpha\neq\alpha',\beta}\phi_\alpha\rangle$$

Localization: the General Theory 2

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- Take $t \to \infty$. Path integral localizes to a neighborhood of the minima of S_{ex}, i.e. 1-loop calculation is exact. Path integral is a sum over the minima of S_{ex}, weighted by a 1-loop determinant factor and by S'.

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- Matter can be chiral or twisted chiral; $SU(2 \mid 1)_A$ contains vector R-symmetry and $SU(2 \mid 1)_B$ contains axial R-symmetry. Mirror symmetry halves the number of distinct possibilities. We're interested in a theory with twisted chiral matter and $SU(2 \mid 1)_A$ symmetry.

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- The bosonic parts of the action are positive definite, so need to study zeroes of S_{ex}.

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- Refinement of argument concerning gauge-fixing in the localization computation.
- Reformulation of proof of $Z = e^{-\mathcal{K}}$ in the language of connections on the conformal manifold.

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- In 2d $\mathcal{N} = (2, 2)$ CFTs, the two-sphere partition function computes the Kähler potential of the conformal manifold.
- Localization enables the *exact* computation of the partition function.
- Outstanding Questions
  - Convergence of the sums in Γ^2.
  - We have ignored possible singular points on the manifold. The study of these could reveal deeper insights about the structure of the space of CFTs.

Acknowledgements

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