

Name: _____

Section: _____

1. Find the general solution to the differential equation

$$y''' - 2y'' + y' - 2y = \cos(2t).$$

Use the method of undetermined coefficients as part of your solution.

$$\begin{aligned} \text{Aux Eq: } r^3 - 2r^2 + r - 2 &= r^2(r-2) + 1(r-2) \\ &= (r^2+1)(r-2) \end{aligned}$$

$$\Rightarrow \text{Gen Homog. Sol: } c_1 \sin t + c_2 \cos t + c_3 e^{2t}$$

Guess for y_p : $A \sin 2t + B \cos 2t$

$$y_p''' = -8A \cos 2t + 8B \sin 2t$$

$$-2y_p'' = 8A \sin 2t + 8B \cos 2t$$

$$+y_p' = 2A \cos 2t - 2B \sin 2t$$

$$-2y_p = -2A \sin 2t - 2B \cos 2t$$

$$y_p''' - 2y_p'' + y_p' - 2y_p = (-6A + 6B) \cos 2t + (6B + 6A) \sin 2t$$

$$\left. \begin{array}{l} -6A + 6B = 1 \\ 6B + 6A = 0 \end{array} \right\} \rightarrow A = -B \quad \left. \begin{array}{l} A = -\frac{1}{12} \\ B = \frac{1}{12} \end{array} \right\} \rightarrow \left. \begin{array}{l} \text{Gen Sol:} \\ c_1 \sin t + c_2 \cos t + c_3 e^{2t} + \frac{1}{12}(-\sin 2t + \cos 2t) \end{array} \right\}$$

2. Solve the system of equations $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix},$$

and

$$\mathbf{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\chi_A(\lambda) = \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$$

$$\lambda = 1: \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \xrightarrow{\sim} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \lambda = -1: \begin{bmatrix} 3 & -1 \\ 3 & 1 \end{bmatrix} \xrightarrow{\sim} \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\text{Gen'l Soln: } \mathbf{y} = c_1 e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{y}(0) = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\sim} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \xrightarrow{\sim} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow{\sim} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \xrightarrow{\sim} \boxed{\mathbf{y}(t) = -\frac{1}{2} e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{2} e^{-t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$