

Name: \_\_\_\_\_

Section: \_\_\_\_\_

1. Find the general solution to the differential equation

$$y''' - 2y'' + y' - 2y = \sin(2t).$$

Use the method of undetermined coefficients as part of your solution.

Aux Eq<sup>n</sup>:  $r^3 - 2r^2 + r - 2 = r^2(r-2) + 1(r-2)$   
 $= (r^2 + 1)(r-2)$

$\Rightarrow$  Gen Homog. Sol<sup>n</sup>:  $c_1 \sin t + c_2 \cos t + c_3 e^{2t}$  (1 pt)

Guess for  $y_p$ :  $A \sin 2t + B \cos 2t$  (1 pt)

$$\begin{aligned} -6A + 6B &= 0 \\ 6B + 6A &= 1 \end{aligned}$$

$$\begin{cases} A = B \\ 12A = 1 \end{cases} \Rightarrow A = B = \frac{1}{12}$$

Gen Sol<sup>n</sup>:

$$c_1 \sin t + c_2 \cos t + c_3 e^{2t} + \frac{1}{12}(\cos 2t + \sin 2t)$$

$$y''' - 2y'' + y' - 2y = (-6A + 6B) \cos 2t + (6B + 6A) \sin 2t$$

2. Solve the system of equations  $\mathbf{y}' = A\mathbf{y}$ , where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix},$$

and

$$\mathbf{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Need to find eigenvectors, evals of  $A$ : (1 pt)

$$\chi_A(\lambda) = \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$$

$$\lambda = 1 : \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = -1 : \begin{bmatrix} 3 & -1 \\ 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Gen<sup>t</sup> Sol<sup>n</sup> :  $\mathbf{y} = c_1 e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$  (1 pt)

$$\mathbf{y}(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} \end{bmatrix} \rightarrow \boxed{\mathbf{y}(t) = \frac{3}{2} e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{2} e^{-t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}} \quad (1 \text{ pt})$$