

Name: _____

Section: _____

1. Find the general solution to the differential equation $y' = Ay$, where

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Aux Eqn: $\lambda^2 - 2\lambda + 2 \rightarrow \lambda = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i$

$$\lambda = 1+i: \begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \xrightarrow{\text{row op}} \begin{bmatrix} 0 & 0 \\ -1 & -i \end{bmatrix} \xrightarrow{\text{row op}} \begin{bmatrix} -i \\ 1 \end{bmatrix} \text{ is eigenvector}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

→ Gen'l Soln: $c_1 e^t \left(\cos t \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \sin t \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right) + c_2 e^t \left(\sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \cos t \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right)$

2. Find the solution of the initial value problem $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = (1, -1)^T$, where

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}.$$

~~$$\text{特征方程 } \chi(\lambda) = (2-\lambda)^2 \rightarrow (A-2I)^2 = 0$$~~

so $e^{At} = e^{2It + (A-2I)t} = e^{2t} (I + t(A-2I))$

$$e^{At} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = e^{2t} \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= e^{2t} \begin{bmatrix} 1+t \\ -1 \end{bmatrix}$$