Quiz #5 MATH 54, Fall 2016, Section 219

Name: ______________________________

Section: ______________________________

1. Consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{R}$ given by $T(f) = \int_0^2 xf(x)dx + f(1)$. Choose bases for $\mathbb{P}_2$ and $\mathbb{R}$ and compute the matrix of T with respect to that basis. Is T onto?

$\mathcal{B} = (1, x, x^2)$ $\quad \mathcal{C} = \{1\}$. $\quad T(1) = \int_0^2 x\,dx + 1 = 2 + 1 = 3$

$T(x) = \int_0^2 x^2 dx + 1 = 11/3$

$T(x^2) = \int_0^2 x^3 dx + 1 = 5 \quad {}_{\mathcal{C}}[T]_{\mathcal{B}} = [3, 11/3, 5]$

2. Let $\mathcal{B} = \{(1, -1), (1, 1)\}$. This is a basis for $\mathbb{R}^2$. Compute the change of basis matrix $P_{\mathcal{B}\leftarrow std}$.

Need to row-reduce

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/2 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right]$$

$$\boxed{\underset{\mathcal{B}\leftarrow std}{P} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}$$

3. With $\mathcal{B}$ as in problem 2, compute $P_{std\leftarrow\mathcal{B}}$.

Need to row reduce

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right]$$; But it's already row reduced!

$$\boxed{\underset{std\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}}$$