

Name: _____

Section: _____

1. Find a basis for the column space of A , where A is the matrix given below. What is the rank of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? (The rank of a linear transformation T is the dimension of the range of T).

$$A = \begin{bmatrix} 2 & -1 & 11 \\ 0 & -2 & 2 \\ 1 & -1 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 6 \\ 0 & -2 & 2 \\ -2 & 1 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 6 \\ 0 & -2 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -1 & 6 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

↑
pivot cols

A basis for $\text{col } A$ is
 $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} \right\}$, so
 $\text{rk}(T) = \dim(\text{col } A) = 2$

2. Consider the basis $B = \{1, 1+x, 1+x+x^2\}$ for \mathbb{P}_2 . Suppose $\mathbf{v} \in \mathbb{P}_2$ is such that $[\mathbf{v}]_B = (1, -1, 0)^T$. What is \mathbf{v} ?

$$[\mathbf{v}]_B = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \text{ means } \mathbf{v} = 1 \cdot 1 + (-1)(1+x) + 0 \cdot (1+x+x^2)$$

$$= 1 - 1 - x = \boxed{-x}$$

3. Find the coordinates of the vector $(0, 1)^T$ with respect to the basis $\{(2, 1)^T, (1, 2)^T\}$ for \mathbb{R}^2 .

Want to solve $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

Thus $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}}$