

Name: _____

Section: _____

1. Find a basis for the column space of A , where A is the matrix given below. What is the rank of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? (The rank of a linear transformation T is the dimension of the range of T).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 2 \\ -1 & 4 & -9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 2 \\ 0 & 6 & -6 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \left| \begin{array}{l} \text{Thus, } \text{rk}(T) = 2 = \dim(\text{col } A) \\ \text{A basis for col } A \text{ is} \\ \{ [1], [-2] \} \end{array} \right.$$

↑
pivot cols

2. Consider the basis $B = \{1, 1+x, 1+x+x^2\}$ for \mathbb{P}_2 . Suppose $\mathbf{x} \in \mathbb{P}_2$ is such that $[\mathbf{x}]_B = (1, 0, 2)^T$. What is \mathbf{x} ?

$$[\mathbf{x}]_{B^2} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \text{ means } \mathbf{x} = 1 \cdot (1) + 0 \cdot (1+x) + 2 \cdot (1+x+x^2)$$

$$= 3 + 2x + 2x^2$$

3. Find the coordinates of the vector $(1, 0)^T$ with respect to the basis $\{(2, 1)^T, (1, 2)^T\}$ for \mathbb{R}^2 .

Want to solve $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}}$$