

Name: \_\_\_\_\_  
 Section: Solutions

1. Compute the inverse of the following matrix by row reduction (it is indeed invertible):

$$\begin{array}{c}
 \left[ \begin{array}{ccc} 3 & -4 & 3 \\ 2 & -4 & 3 \\ -2 & 3 & -2 \end{array} \right] \\
 \left[ \begin{array}{cccc|cc} 2 & -4 & 3 & 0 & 1 & 0 \\ 3 & -4 & 3 & 1 & 0 & 0 \\ -2 & 3 & -2 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|cc} 2 & -4 & 3 & 0 & 1 & 0 \\ 0 & 2 & -\frac{3}{2} & 1 & -\frac{3}{2} & 0 \\ 0 & -1 & 1 & 0 & 1 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|cc} 2 & -4 & 3 & 0 & 1 & 0 \\ 0 & 1 & -\frac{3}{2} & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 1 & \frac{1}{2} & 2 \end{array} \right] \\
 \rightarrow \left[ \begin{array}{cccc|cc} 2 & -4 & 3 & 0 & 1 & 0 \\ 0 & 1 & -\frac{3}{2} & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & 2 & 1 & 4 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|cc} 2 & 4 & 0 & -6 & -2 & -12 \\ 0 & 1 & 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 & 4 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|cc} 2 & 0 & 0 & 2 & -2 & -20 \\ 0 & 1 & 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 & 4 \end{array} \right] \\
 \rightarrow A^{-1} = \left[ \begin{array}{ccc} 1 & -1 & 0 \\ 2 & 0 & 3 \\ 2 & 1 & 4 \end{array} \right]
 \end{array}$$

2. Is the linear transformation that gives rise to the following transformation one-to-one?  
 Onto? Do not use row reduction.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$2 \cdot \text{Row 1} + \text{Row 2} + 0 \cdot \text{Row 3} = 0 \Rightarrow$  rows are L.D.  
 $\Rightarrow$  not onto

$2 \cdot \text{col 1} - \text{col 2} + 0 \cdot \text{col 3} = 0 \Rightarrow$  cols are L.D.  
 $\Rightarrow$  not 1-1

3. Provide an example or explain why there exists of a  $2 \times 3$  matrix  $A$  and a  $3 \times 2$  matrix  $C$  such that  $CA = I_3$ . If an example exists, try to give the simplest possible example you can find.

$CA = I_3 \Rightarrow Ax = 0$  has no non-zero sol<sup>n</sup>s

$\Rightarrow$  a REF for  $A$  has a pivot in every col.

But  $A$  can have at most 2 pivots, so it can't have a pivot in every one of its 3 columns.