

Name: _____
 Section: Solutions

1. Compute the inverse of the following matrix by row reduction (it is indeed invertible):

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 100 \\ 2 & 0 & 3 & 010 \\ 2 & 1 & 4 & 001 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & -1 & 0 & 100 \\ 0 & 2 & 3 & -210 \\ 0 & 3 & 4 & -201 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & -1 & 0 & 100 \\ 0 & 2 & 3 & -210 \\ 0 & 0 & -2 & 1-\frac{3}{2}1 \end{bmatrix}$$

$$\xrightarrow{\quad} \begin{bmatrix} 1 & -1 & 0 & 100 \\ 0 & 2 & 3 & -210 \\ 0 & 0 & 1 & -232 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & -1 & 0 & 100 \\ 0 & 2 & 4 & -86 \\ 0 & 0 & 1 & -232 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & -4 & 3 \\ 0 & 0 & 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ 2 & -4 & 3 \\ -2 & 3 & -2 \end{bmatrix}$$

2. Is the linear transformation that gives rise to the following transformation one-to-one?
 Onto? Do not use row reduction.

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Col 2 - 2 Col 1 + 0 Col 3 = 0, so the cols
 are L.D. The transformation is therefore not 1-1.
transformation is
 Since the matrix is square, it is also not onto, by
 the invertible matrix theorem.

3. Provide an example or explain why none exists of a 2×3 matrix A and a 3×2 matrix C such that $AC = I_2$. If an example exists, try to give the simplest possible example you can find.

$AC = I_2 \Rightarrow A$ has L.I. rows. Simplest matrix with L.I. rows: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Cols of C should be solutions of $Ax = e_i \in \mathbb{C}^3$

$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, so can take

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$