Quiz #8 MATH 54, Fall 2016, Section 224

Name:

Section: Solutions

1. Let

$$A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix}.$$

Using the fact that $\mathbf{v}_1 = (-2, 2, 1)^T$ and $\mathbf{v}_2 = (1, 1, 0)^T$ are eigenvectors for A, find an orthogonal basis for $\mathbb{R}^3$ consisting of eigenvectors for A. (Hint: You don't need to solve the characteristic equation or find eigenspaces in this problem.)

See the Solutions for other section's quiz.

2. Let $\mathcal{A} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be two bases for a vector space V, and suppose

$$\mathbf{u}_1 = \mathbf{v}_1 - 2\mathbf{v}_2 + 3\mathbf{v}_3, \quad \mathbf{u}_2 = -\mathbf{v}_1 + \mathbf{v}_2, \quad \mathbf{u}_3 = 4\mathbf{v}_2 - 6\mathbf{v}_3.$$

Let $\mathbf{v} = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3$. Find $[\mathbf{v}]_{\mathcal{A}}$.

$$\underline{v} = \underline{v_1} - 2\underline{v_2} + 3\underline{v_3} - \underline{v_1} + \underline{v_3} + 4\underline{v_2} - 6\underline{v_3}$$

$$= 0\underline{v_1} + 3\underline{v_2} - 3\underline{v_3} \Rightarrow \boxed{[\underline{v}]_{\mathcal{A}} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}}$$