Name: _____

Section: __Solutions__

1. Determine whether or not the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ given below are linearly independent. Use row reduction.

Can put the $\underline{v}$'s as
rows or columns

$$\mathbf{v}_1 = (1, 0, 1, 0)$$
$$\mathbf{v}_2 = (1, -2, 1, 2)$$
$$\mathbf{v}_3 = (0, 1, 2, 1)$$
$$\mathbf{v}_4 = (3, 1, 9, 5)$$

Not a pivot in every row, so L.D.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -2 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 3 & 1 & 9 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 6 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 6 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. For which values of a, b, and c does the following system of equations have a solution? Use row reduction. (Hint: You should find an equation that a, b, and c need to satisfy for the system to be consistent.)

$$\begin{cases} x + y = a \\ 2x - 3y = b \\ 4x - y = c \end{cases}$$

Need $c - b - 2a = 0$

$$\begin{bmatrix} 1 & 1 & a \\ 2 & -3 & b \\ 4 & -1 & c \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & a \\ 0 & -5 & b-2a \\ 0 & -5 & c-4a \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & a \\ 0 & -5 & b-2a \\ 0 & 0 & c-b-2a \end{bmatrix}$$

3. True/False: Circle the appropriate choice for each question. Circle True only if the statement is always true, and false if the statement is at least sometimes false.

(True) False   If a matrix has more columns than rows, then its columns are linearly dependent.

True (False)   A set of vectors containing the zero vector can be linearly independent.