

Quiz #2 MATH 54, Fall 2016, Section 219

Name: \_\_\_\_\_  
 Section: Solutions

1. Determine whether or not the vectors  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$  given below are linearly independent.  
 Use row reduction.

Can put the  $\mathbf{v}_i$ 's as  
 rows  $\rightarrow$  columns.

$$\mathbf{v}_1 = (1, 0, 1, 0)$$

$$\mathbf{v}_2 = (1, -2, 1, 2)$$

$$\mathbf{v}_3 = (0, 1, 2, 1)$$

$$\mathbf{v}_4 = (2, 1, 9, 5)$$

Pivot in every  
 row so L.I.

$$\left[ \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 1 & -2 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 2 & 1 & 9 & 5 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - R_1 \\ R_4 \rightarrow R_4 - 2R_1}} \left[ \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 7 & 5 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow R_3 + \frac{1}{2}R_2 \\ R_4 \rightarrow R_4 + \frac{1}{2}R_2}} \left[ \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 7 & 6 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 7 & 6 \end{array} \right]$$

2. For which values of  $a$ ,  $b$ , and  $c$  does the following system of equations have a solution?  
 Use row reduction. (Hint: You should find an equation that  $a$ ,  $b$ , and  $c$  need to satisfy for  
 the system to be consistent.)

$$\begin{cases} x + y = a \\ 2x - 3y = b \\ 6x - 4y = c \end{cases}$$

Need  $c - 2b - 2a = 0$

$$\left[ \begin{array}{ccc} 1 & 1 & a \\ 2 & -3 & b \\ 6 & -4 & c \end{array} \right] \xrightarrow{\quad} \left[ \begin{array}{ccc} 1 & 1 & a \\ 0 & -5 & b-2a \\ 0 & -10 & c-6a \end{array} \right] \xrightarrow{\quad} \left[ \begin{array}{ccc} 1 & 1 & a \\ 0 & -5 & b-2a \\ 0 & 0 & c-2b-2a \end{array} \right]$$

3. True/False: Circle the appropriate choice for each question. Circle True only if the statement is always true, and false if the statement is at least sometimes false.

True  False If a matrix has more rows than columns, then  $Ax = \mathbf{b}$  has a solution for all  $\mathbf{b}$ .  
 True  False If two vectors are linearly independent, then one is a multiple of the other.