

Name: _____

Section: Solutions

1. Use row reduction to solve the following system of linear equations, or show that there are no solutions.

$$\begin{cases} x + 2y + z = 8 \\ 2x - 3y + 4z = 4 \\ 3x - y + 4z = 11 \end{cases}$$

$x=3, y=2, z=1$

$$\begin{bmatrix} 1 & 2 & 1 & 8 \\ 2 & -3 & 4 & 4 \\ 3 & -1 & 4 & 11 \end{bmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1}} \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -7 & 2 & -12 \\ 0 & -7 & 1 & -13 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - R_2} \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -7 & 2 & -12 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_2 \rightarrow R_2 + 2R_3 \\ R_1 \rightarrow R_1 + R_3}} \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & -7 & 0 & -14 \\ 0 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 + \frac{7}{2}R_2} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & -7 & 0 & -14 \\ 0 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{\substack{R_2 \rightarrow \frac{1}{2}R_2 \\ R_3 \rightarrow -R_3}} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

2. Put the following matrix in reduced row echelon form. If this were the augmented matrix of a linear system, how many free variables would the solution have?

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 1 & -1 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 1 & -1 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & -2 & 0 & -8 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 + \frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & 1 & 8 \\ 0 & -2 & 0 & -8 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightarrow -\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & 1 & 8 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

1 free variable¹