Worksheet 9: February 21

1 Modular Inverses

1. For each of the following congruences $a \mod b$, determine whether a is invertible mod b, and if so, find its modular inverse.

   (a) $5 \mod 7$

   (b) $12 \mod 26$

   (c) $3 \mod 10$

   (d) $59 \mod 11$

   (e) $15 \mod 18$

   (f) $45 \mod 46$

2 Chinese Remainder Theorem

2. For each of the following sets of congruences, find some integer x such that every congruence holds.

   (a) $\begin{cases} x \equiv 2 \mod 3 \\ x \equiv 1 \mod 4 \end{cases}$

   (b) $\begin{cases} x \equiv 1 \mod 4 \\ x \equiv 0 \mod 5 \\ x \equiv 4 \mod 7 \end{cases}$

   (c) $\begin{cases} x \equiv 1 \mod 2 \\ x \equiv 2 \mod 3 \\ x \equiv 6 \mod 13 \end{cases}$

   (d) $\begin{cases} x \equiv 75 \mod 457 \\ x \equiv 75 \mod 6781 \end{cases}$

3 Fermat's Little Theorem

3. Evaluate the following congruences:

   (a) $2^{44} \mod 7$

   (b) $6^{123} \mod 11$

   (c) $26^{90941} \mod 13$

   (d) $43^{43} \mod 11$

4. State and prove Fermat's Little Theorem.