Worksheet 8: February 14

1 Divisors

1. List all divisors of each of the following numbers.

   (a) 6

   (b) 13

   (c) 18

   (d) 25

   (e) 81

   (f) 100

2. From the previous question: For each of the following pairs of integers, find their greatest common divisor. Which pairs are *relatively prime*?

   (a) 13 and 25

   (b) 6 and 81

   (c) 25 and 100

   (d) 6 and 25

   (e) 18 and 81

2 The Euclidean algorithm and the Chinese remainder theorem

3. (a) Use the Euclidean algorithm to find the GCD of 270 and 192.

   (b) Let d be the GCD you just found. Work through the steps of the algorithm to find integers a and b such that $270a + 192b = d$.

4. (a) Use the Euclidean algorithm to find the GCD of 13 and 98.

   (b) Let d be the GCD you just found. Work through the steps of the algorithm to find integers a and b such that $13a + 98b = d$.

5. For each of the following sets of congruences, find *all* integers x such that every congruence holds.

   (a) $\begin{cases} x \equiv 2 \mod 5 \\ x \equiv 3 \mod 7 \end{cases}$

   (b) $\begin{cases} x \equiv 1 \mod 2 \\ x \equiv 2 \mod 3 \\ x \equiv 3 \mod 5 \end{cases}$

   (c) $\begin{cases} x \equiv 8 \mod 13 \\ x \equiv 76 \mod 98 \end{cases}$

   (d) $\begin{cases} x \equiv 1 \mod 6 \\ x \equiv 2 \mod 10 \end{cases}$