Worksheet 7: February 12 (Solutions)

1 Divisibility and Modularity

1. For each of the following numbers n, calculate $n \mod 15$. Which of these numbers are divisible by 15? Which of them are congruent to each other mod 15?

   (a) 0 is divisible by 15

   (b) $3 \equiv 3 \mod 15$

   (c) 15 is divisible by 15

   (d) -450 is divisible by 15

   (e) $1000 \equiv 10 \mod 15$

   (f) $-1000 \equiv 5 \mod 15$

   (g) 30165 is divisible by 15

   (h) $30168 \equiv 3 \mod 15$

   0, 15, -450, and 30165 are equivalent mod 15. 3 and 30168 are equivalent mod 15.

2. Calculate the following:

   (a) $(10^6 - 37^8 + 561^4 - 77852^{36}) \mod 2 \equiv (0^6 - 1^8 + 1^4 - 0^{36}) \mod 2 \equiv 0 \mod 2$

   (b) $20^5 \mod 21 \equiv (-1)^5 \mod 21 \equiv (-1) \mod 21 \equiv 20 \mod 21$

   (c) $(9999 \times 15 - 1234) \mod 10 \equiv (9 \times 5 - 4) \mod 10 \equiv 1 \mod 10$

3. Suppose a, b, c are integers such that $a \equiv b \mod 7$. Prove or disprove the following:

   (a) $a^c \equiv b^c \mod 7$
   True by induction

   (b) $c^a \equiv c^b \mod 7$
   False. $2^0 \equiv 1 \mod 7$ but $2^7 = 128 \equiv 2 \mod 7$

   (c) $(a \mod c) \equiv (b \mod c) \mod 7$
   False. $(0 \mod 6) \mod 7 \equiv 0 \mod 7$ but $(7 \mod 6) \mod 7 \equiv 1 \mod 7$

4. Convert the following base-10 integers to binary.

   (a) $56 = 111000_2$

   (b) $184 = 10111000_2$

   (c) $255 = 11111111_2$

   (d) $4532 = 1000110110100_2$