Worksheet 7: February 12

1 Divisibility and Modularity

1. For each of the following numbers n, calculate $n \mod 15$. Which of these numbers are divisible by 15? Which of them are congruent to each other mod 15?

   (a) 0

   (b) 3

   (c) 15

   (d) -450

   (e) 1000

   (f) -1000

   (g) 30165

   (h) 30168

2. Calculate the following:

   (a) $(10^6 - 37^8 + 561^4 - 77852^{36}) \mod 2$

   (b) $20^5 \mod 21$

   (c) $(9999 \times 15 - 1234) \mod 10$

3. Suppose a, b, c are integers such that $a \equiv b \mod 7$. Prove or disprove the following:

   (a) $a^c \equiv b^c \mod 7$

   (b) $c^a \equiv c^b \mod 7$

   (c) $(a \mod c) \equiv (b \mod c) \mod 7$

4. Convert the following base-10 integers to binary.

   (a) 56

   (b) 184

   (c) 255

   (d) 4532