Worksheet 6: February 7

1 Cardinality

1. For each of the following sets A, determine whether A is finite, countable, or uncountable. Provide an (informal) proof for each.

   (a) $A = \{2, 6, 19\}$

   (b) $A = \mathbb{Z}$

   (c) $A = \{\text{prime numbers}\}$

   (d) $A = \mathbb{Q}$

   (e) $A = \{\text{atoms in the universe}\}$

   (f) $A = \mathbb{Z} \times \mathbb{Z}$

   (g) $A = \mathbb{R}$

   (h) $A = \{\text{subsets of } \mathbb{Z}\}$

2. For each of the following pairs of sets, determine which has larger cardinality. Find an injective map from the smaller map to the larger one. If they have the same cardinality, try to find a bijective map between them.

   (a) $A = \{0, 2, 5\}$, $B = \{1, 3\}$

   (b) $A = \{\text{submitted Math 55 Homework 1s}\}$, $B = \{\text{students in Math 55}\}$

   (c) $A = \{\text{people in the world}\}$, $B = \{\text{human brain cells}\}$

   (d) $A = \{\text{books}\}$, $B = \{\text{authors}\}$

   (e) $A = \{\text{positive integers}\}$, $B = \{\text{positive even integers}\}$

   (f) $A = \{\text{positive integers}\}$, $B = \{\text{prime numbers}\}$

   (g) $A = \mathcal{P}(\{1, 2, \ldots, n\})$, $B = \{\text{binary strings of length } n\}$

   (h) $A = \mathbb{R}$, $B = \mathbb{Z}$

   (i) $A = \mathbb{Q}, B = \mathbb{Q} \times \{0, 1, 2\}$

   (j) $A = \mathbb{Z}$, $B = \mathbb{Z} \times \mathbb{Z}$

   (k) $A = \mathbb{Z} \times \mathbb{Z}$, $B = \mathbb{Q}$

   (l) $A = \mathbb{Z}$, $B = \mathcal{P}(\mathbb{Z})$