Worksheet 5: February 5 (Solutions)

1 Sets, part 2

1. Determine the truth value of the following statements. Give a brief proof for each.

   (a) $\forall A : A \cap \overline{A} = \emptyset$
   True. No element can be in both A and $\overline{A}$.

   (b) $\forall A : A \cup \overline{A} = U$
   True. Every element is in either A or $\overline{A}$.

   (c) $\forall A \forall B : A \setminus B = A \cap \overline{B}$
   True. Definition of the $\setminus$ operation.

   (d) $\forall A \forall B : (A \setminus B) \cup (A \setminus \overline{B}) = A$
   True. Every element of A is either not in B or not in $\overline{B}$.

   (e) $\forall A \forall B : (A \setminus B) \cup (B \setminus A) \cup (A \cap B) = A \cup B$
   True. Try drawing a Venn diagram.

2. What is the power set of $A = \{2, 3, 5\}$?
   $\{\emptyset, \{2\}, \{3\}, \{5\}, \{2, 3\}, \{2, 5\}, \{3, 5\}, \{2, 3, 5\}\}$

2 Functions

3. Determine which of the following functions are *injective* (one-to-one), *surjective* (onto), or *bijective*.

   (a) $f : \mathbb{R} \to \mathbb{R},\ f(x) = x^3$
   Bijective.

   (b) $f : \mathbb{Z} \to \mathbb{Z},\ f(x) = x^3$
   Injective.

   (c) $f : \{0, 1\} \to \mathbb{Z},\ f(x) = 5$
   Neither.

   (d) $f : \mathbb{Z} \to \{0, 1, 2, 3\},\ f(x) = $ (the remainder of x divided by 4)
   Surjective.

   (e) $f : \mathbb{R} \times \mathbb{R} \to \mathbb{C},\ f(x, y) = x + iy$
   Bijective.

4. Let $f : A \to B$ and $g : B \to C$. What can you say about the injectivity and surjectivity of $g \circ f$, given that...

   (a) ...f is injective and g is injective?
   $g \circ f$ is injective.

   (b) ...f is injective and g is surjective?
   Nothing.

   (c) ...f is surjective and g is injective?
   Nothing.

   (d) ...f is surjective and g is surjective?
   $g \circ f$ is surjective.

   (e) ...f is bijective and g is injective?
   $g \circ f$ is injective.

   (f) ...f is surjective and g is bijective?
   $g \circ f$ is surjective.

   (g) ...f is bijective and g is bijective?
   $g \circ f$ is bijective.