

Worksheet 5: February 5

1 Sets, part 2

1. Determine the truth value of the following statements. Give a brief proof for each.

(a) $\forall A : A \cap \bar{A} = \emptyset$

(b) $\forall A : A \cup \bar{A} = U$

(c) $\forall A \forall B : A \setminus B = A \cap \bar{B}$

(d) $\forall A \forall B : (A \setminus B) \cup (A \setminus \bar{B}) = A$

(e) $\forall A \forall B : (A \setminus B) \cup (B \setminus A) \cup (A \cap B) = A \cup B$

2. What is the power set of $A = \{2, 3, 5\}$?

2 Functions

3. Determine which of the following functions are *injective* (one-to-one), *surjective* (onto), or *bijective*.

(a) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3$

(b) $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = x^3$

(c) $f : \{0, 1\} \rightarrow \mathbb{Z}, f(x) = 5$

(d) $f : \mathbb{Z} \rightarrow \{0, 1, 2, 3\}, f(x) = (\text{the remainder of } x \text{ divided by } 4)$

(e) $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{C}, f(x, y) = x + iy$

4. Let $f : A \rightarrow B$ and $g : B \rightarrow C$. What can you say about the injectivity and surjectivity of $g \circ f$, given that...

(a) ... f is injective and g is injective?

(b) ... f is injective and g is surjective?

(c) ... f is surjective and g is injective?

(d) ... f is surjective and g is surjective?

(e) ... f is bijective and g is injective?

(f) ... f is surjective and g is bijective?

(g) ... f is bijective and g is bijective?