Worksheet 19: April 8

Principles to Remember

- Remember the following definitions: **experiment**, **sample space**, **event**, **probability distribution**, **random variable**, **expected value**, **independence**.
- **Variance:** The variance of a random variable X defined on a sample space S is defined as $V(X) = \sum_{s \in S} p(s)(X(s) - E(X))^2$, which is equivalent to $E((X - E(X))^2)$ *(why?)*.
- Properties of variance:
  1. $V(aX) = a^2 V(X)$ for any $a \in \mathbb{R}$ and any random variable X
  2. $V(X + a) = V(X)$ for any $a \in \mathbb{R}$ and any random variable X
  3. $V(X + Y) = V(X) + V(Y)$ for ***independent*** random variables X and Y
- **Chebyshev's inequality:** If X is a random variable on a sample space S, and r is a positive real number, then $p(|X(s) - E(X)| \geq r) \leq V(X)/r^2$.

Exercises

1. Let X be the value that comes up when a fair die is rolled. Find $V(X)$.

2. What is the variance of the number of heads that come up when a fair coin is flipped ten times?

3. Prove $V(X) = E(X^2) - E(X)^2$. (This formula is often a more convenient way of finding the variance; know it!)

4. Prove the three properties of variance listed above in "Principles to Remember."

5. Prove that independence is required for the third property: Describe two non-independent random variables X and Y such that $V(X+Y) \neq V(X) + V(Y)$.

6. Let F be the event that when flipping n coins, the number of tails that comes up deviates from the mean by more than $5\sqrt{n}$. Use Chebyshev's Inequality to find an upper bound on $p(F)$.

7. Suppose that a recycling center recycles an average of 50,000 aluminum cans a day, with a variance of 10,000 cans. Use Chebyshev's Inequality to provide a lower bound on the probability that the center will recycle between 40,000 and 60,000 cans on a certain day.