Worksheet 18: April 3

Principles to Remember

- Remember what an **experiment**, **sample space**, **event**, and **probability distribution** are.
- Know the following terms:
  - **random variable:** A *function* from a sample space to $\mathbb{R}$.
  - **expected value:** The EV of a random variable X on a sample space S is defined as $E(X) = \sum_{s \in S} p(s)X(s)$.
- **Independence:** Two RV's X and Y on S are independent if

$$p(X = s \wedge Y = s) = p(X = s)p(Y = s) \quad \forall s \in S.$$

- Linearity of expected value:
  - $E(X + Y) = E(X) + E(Y)$ for **any** random variables X and Y
  - $E(XY) = E(X)E(Y)$ for **independent** random variables X and Y
  - $E(aX + b) = aE(X) + b$ for $a, b \in \mathbb{R}$ and any random variable X

Exercises

1. Consider rolling a fair die 4 times.
   (a) What is the expected number of 6's that appear?
   (b) What is the expected sum of the numbers that appear?

2. Repeat the previous problem, but now consider a biased die, where 6 comes up twice as often as any other number.

3. Suppose that we flip a fair coin until a tails comes up or we flip it 5 times. What is the expected number of times that we flip the coin?

4. Suppose that we flip a fair coin until a tails comes up. What is the expected number of times that we flip the coin?

5. The final exam of a discrete mathematics course consists of 50 true/false questions, each worth two points, and 25 multiple choice questions, each worth 4 points. The probability that Linda answers a true/false question correctly is 0.9, and the probability that she answers a multiple choice question correctly is 0.8. What is her expected score on the final?

6. Suppose X and Y are random variables in a sample space S, and $X(s)$ and $Y(s)$ are nonnegative for all $s \in S$. Let Z be the random variable defined by $Z(s) = \max\{X(s), Y(s)\}$. Show that $E(Z) \leq E(X) + E(Y)$.