Worksheet 15: March 13

Principles to Remember

- **Permutations with Repetitions:** The number of r-permutations from a set with n elements when repetition is allowed is n^r.

- **Generalized Permutations:** If we have n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ..., and n_k indistinguishable objects of type k, and $n = n_1 + n_2 + \cdots + n_k$, then the number of permutations of all these objects is $\dfrac{n!}{n_1!\, n_2! \cdots n_k!}$. This is known as the **multinomial coefficient** $\dbinom{n}{n_1, n_2, \ldots, n_k}$.

- **Combinations with Repetitions:** The number of r-combinations from a set with n elements when repetition is allowed is $\dbinom{n+r-1}{r}$.

Exercises

1. How many permutations are there of the word 'COMBINATORICS'?

2. How many ways are there to line up three apples, four bananas, five oranges, and six kiwis?

3. How many strings of 10 ternary digits (0, 1, or 2) contain exactly three 0's, five 1's, and two 2's?

4. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 11$, where x_1, x_2, x_3, x_4 are...

   (a) nonnegative integers?
   (b) positive integers?
   (c) integers?

5. Given a standard 52-card deck and four players, how many ways are there to deal a hand of 13 cards to each player?

6. How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

7. How many ways are there to distribute...

   (a) n distinguishable balls into k distinguishable boxes?
   (b) n indistinguishable balls into k distinguishable boxes?

8. How many ways can n books be placed on k distinguishable shelves...

   (a) if the books are n identical copies of Kenneth H. Rosen's *Discrete Mathematics and Its Applications*?
   (b) if no two books are the same, but the positions of the books on the shelves don't matter (only which shelf they're placed on)?
   (c) if no two books are the same, and the positions of the books on the shelves matter?