# Worksheet 12: March 4

## 1 Counting

### Principles to Remember

- Product Rule: A sequence of two tasks, one of which can be performed in $n_1$ ways and the other of which can be performed in $n_2$ ways, can be performed in a total of $n_1 n_2$ ways.
- Sum Rule: A single task, which can be performed in $n_1$ ways or in $n_2$ other ways, can be performed in a total of $n_1 + n_2$ ways.
- Inclusion-Exclusion: A single task, which can be performed in $n_1$ ways or $n_2$ ways (not necessarily distinct), where $v$ ways are counted twice, can be performed in $n_1 + n_2 - v$ ways.

### Exercises

1. How many license plates can be formed from seven characters, where each character is either a letter or a number?

2. Let $S = [10]$ (shorthand for the set $\{1, 2, \ldots, 10\}$).
   (a) How many subsets of $S$ are there?
   (b) How many subsets of $S$ are there containing 1? Containing 10?
   (c) How many subsets of $S$ contain both 1 and 10?
   (d) How many subsets of $S$ contain at least one of 1 or 10?
   (e) How many subsets of $S$ contain neither 1 nor 10?

3. How many functions from $[m]$ to $[n]$ are there...
   (a) ...in total?
   (b) ...which are injective (1-1)?
   (c) ...which are surjective (onto)?

4. How many three-letter initials can people have if...

   (a) Letters cannot be repeated?

   (b) The first initial must be A?

   (c) Both of the above?

5. How many positive integers between 500 and 1000...

   (a) ...are divisible by 7?

   (b) ...are divisible by both 7 and 11?

   (c) ...are divisible by 7 but not by 11?

   (d) ...have distinct digits?

   (e) ...have distinct digits and are even?

6. How many injective functions are there from a set with 5 elements to a set with...

   (a) 4 elements?

   (b) 5 elements?

   (c) 6 elements?

   (d) 7 elements?

7. How many subsets of [100] have more than one element?

8. How many palindromes are there among the bit strings of length $n$?