Worksheet 10: February 26

1 A Few More Words on Fermat

1. State and prove Fermat's Little Theorem (I really want you to be able to do this!)

2. Evaluate the following congruences:
   (a) $7^{1462} \mod 11$
   (b) $19^{603} \mod 7$
   (c) $34^{567} \mod 17$

2 Induction

3. Prove that for any $n \in \mathbb{Z}^+$, $1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$ (the n-th *triangular number*).

4. Prove that for any $n \in \mathbb{Z}^+$, $1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ (the n-th *square pyramidal number*).

5. Prove that for any $n \in \mathbb{Z}^+$, $\dfrac{1}{2} + \dfrac{1}{2^2} + \cdots + \dfrac{1}{2^n} = 1 - \dfrac{1}{2^n}$

6. Consider the following inductive "proof" that all horses are the same color.

   > Let $P(n)$ be the statement that all groups of n horses are the same color. Clearly $P(1)$ is true, because if you only have one horse then all the horses you have are the same color. In the inductive step, suppose that $P(n)$ is true. Then if you have $n + 1$ horses, the first n are all the same color, and the last n are the same color. The $n - 1$ horses shared between these two groups must all be the same color, so the first and last horse must also be the same color, and therefore all $n+1$ horses are the same color. Therefore $P(n) \to P(n + 1)$ is true for all n, and since we have the base case $P(1)$, we have that $P(n)$ is true for all n.

   Why is this wrong?