Chapter 10.2-3: Graph Isomorphism and Connectedness

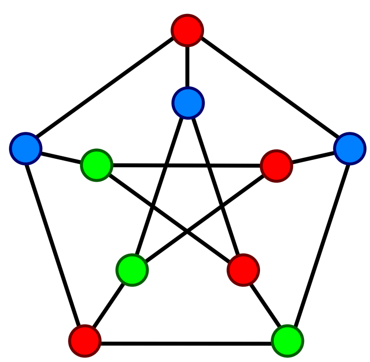
Wednesday, August 5

Summary

- An isomorphism from $G = (V, E)$ to $H = (W, F)$ is a bijection $\varphi : V \to W$ such that $(u, v) \in E$ if and only if $(\varphi(u), \varphi(v)) \in F$.
- An automorphism is an isomorphism from G to itself.
- A walk is a sequence of edges such that successive edges share a common vertex.
- A path is a walk with no repeated vertices.
- A cycle is a path that ends where it began.
- A trail is a walk with no repeated edges.
- A graph is connected if any two vertices are connected by a path.
- The distance between two vertices is the length of the shortest path connecting them.
- The diameter of a graph is the maximum distance between two vertices.

Isomorphisms

1. (★) Show that C_5 and $\overline{C_5}$ are isomorphic.
2. (★) Show that C_4 and $\overline{C_4}$ are not isomorphic.
3. Show that if G has n vertices and G is isomorphic to $\overline{G}$ then $n \equiv 0 \pmod 4$ or $n \equiv 1 \pmod 4$ (Hint: count edges).
4. (★) Find a graph G on 4 vertices such that G and $\overline{G}$ are isomorphic.
5. (★) How many automorphisms are there on C_n? K_n?
6. Find all nonisomorphic graphs with 4 vertices.
7. (Challenge) Show that the Petersen graph, shown below, has 120 automorphisms.



Paths and Connectedness

1. (★) Find the diameter of $P_n, C_n, K_n, K_{m,n}$.

2. If v has odd degree in G then there is some w of odd degree such that v and w are connected by a path.

3. Find all non-isomorphic trees with 6 vertices.

4. Find a graph with n vertices, $n-1$ edges, and diameter 2.

5. (★) Count the number of 4-cycles in $K_{m,n}$.

6. Prove that $d(x,y) + d(y,z) \geq d(x,z)$ for any vertices $x, y, z \in G$. Find an example of a graph and 3 vertices in the graph where the two sides are not equal.

7. Prove: If every vertex in a graph G has degree at least 2 then then G contains a cycle.

Suggested From Rosen

10.3: 34-44, 45-46, 53-56, 66
10.4: 19-25, 45, 64