Chapter 6.3: Permutations and Combinations

Tuesday, July 21

Summary

- $P(n, k) =$ "k-permutations of n" $= n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$
- $\binom{n}{k} =$ "n choose k" $= \frac{n(n-1)\cdot(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$
- $\binom{n}{k} = \binom{n}{n-k}$
- Combinatorial Proofs 1: If A and B are finite sets and $f : A \to B$ is a bijection, then $|A| = |B|$.
- Combinatorial Proofs 2: Any two (correct) ways of counting the elements in a set will yield the same answer.

Warmup

1. (★) Six people are in a club. How many ways to choose three executive members? How many ways are there to choose a president and two vice presidents? Why is the second number divisible by the first?
2. Ten people are in a room, and everyone shakes everyone else's hand exactly once. How many handshakes were there?

Inclusion-Exclusion

1. How many 5-digit numbers start with a 1 or end with a 0 (or both)?
2. (★) How many 5-digit numbers contain at least one 1 and at least one 3? (Hint: what is the complemet of this set in the set of 5-digit numbers?)

Tricks With Counting

1. A coin is flipped seventy-three times. How many possible outcomes have more heads than tails? (Hint: what happens if you swap the words "heads" and "tails"?)
2. How may ways are there to place four red balls and two blue balls around a circular table?
3. (★) How many distinct permutations of the letters CHECKMATE contain the string TEAM? (Hint: treat the string "TEAM" as a solid block and go from there.)

Combinatorial Proofs

1. People go around in a room shaking hands with one another (though it is not necessarily true that everybody shakes everybody else's hand). Prove that the number of people who shake an *odd* number of hands is *even*. (Count the number of person-handshakes, first by person and then by handshake.)

2. ($\bigstar$) Find an argument by bijection that the number of subsets of $\{1 \dots n\}$ of even size is equal to the number of subsets of odd size. (Is 1 in your set, or isn't it?)

3. Find an argument by bijection that $\binom{2n}{n}$ is even for all $n \geq 1$. (Let S be the set of all $n-$combinations of $\{1 \dots 2n\}$ and find a bijection from S to itself. If no sets are left fixed under this function, why does that prove that the size is even?)

Suggested From Rosen

6.3: 19, 21, 23, 24, 33, 34, 35