

# Chapters 6.1-6.2: Counting

Monday, July 20

How many three-letter initials can people have?

If a password consists of a two-digit number followed by a five-character string of English letters, how many possible passwords are there?

A palindrome is a string that reads the same forwards and backwards (e.g. “1771”) How many 4-digit numbers are palindromes? How many 5-digit numbers?

A multiple choice test has 6 questions with 4 possible answers each. Students taking the test also have the option to leave answers blank. How many students must take the test in order to guarantee that at least two students have the exact same answers?

How many numbers between 1 and 1000

1. Are divisible by both 7 and 11?
2. Are divisible by either 7 or 11?
3. Are divisible by 7 but not by 11?
4. Are divisible by neither 7 nor 11?

36 students go to a hot dog stand and order hot dogs. Every student orders at least one topping. You have the following information about their topping choices:

- |                                      |                                           |
|--------------------------------------|-------------------------------------------|
| 1. 18 ask for mustard.               | 5. 31 ask for onions or relish (or both). |
| 2. 21 ask for onions.                | 6. 17 ask for exactly two toppings.       |
| 3. 18 ask for relish.                |                                           |
| 4. 8 ask for mustard but not onions. | 7. 2 ask for all three toppings.          |

How many students order exactly one topping? (Try making a Venn diagram. See also Chapter 8.5 of Rosen)

Use a tree diagram to find the number of subsets of  $\{3, 7, 9, 11, 24\}$  such that the sum of the elements in the subset is less than 28.

If we make a 4-sided die out of a tetrahedron (4 faces, all equilateral triangles), then how many possible arrangements of the numbers are there? How many possible arrangements are there on a 6-sided die?

Put an upper bound on the number of four-letter words in the English language. Try to make this bound as tight as possible by introducing rules about what letters words must have and what letter combinations are not possible. Try any of the following:

1. Every word must have at least one vowel.
2. “q” must always be followed by “u,” then some vowel (with rare exceptions).
3. If a word begins with “m,” “n,” “l,” “r,” or “h,” the next letter must be a vowel.

(Harder) Let  $S$  be any positive integer. Use the Pigeonhole Principle to show that if we write the numbers  $S, 2S, 3S, \dots, 10S$  one underneath the other, then in every full column there will be at least one 0 or 9 appearing.