Sums

What is a closed form for $\sum_{k=1}^{n}(-1)^k k^2$? Find the first few values, then come up with a conjecture.

Find (at least) two ways to express the sum of the triangle below:

```
1
2   1
3   3    1
4   5    4    1
5   7    7    5    1
6   9    10   9    6    1
```

Put in summation notation: what is the x^2 coefficient of $(a_1 x + a_0)(b_1 x + b_0)(c_1 x + c_0)$? What is the x^2 coefficient of $(x+1)^3$?

Here is a board, with some X's on it:

			X				
		X				X	
X	X						
							X
					X		
		X	X				
				X			
	X					X	

Each square is worth some number of points: for every X it shares a row with (including the square it is on), it gets a point. For every X it shares a column with (including the square it is on), it *loses* a point. What is the sum of the point values of all the squares on the board? (Hint: you have to sum over the right variable. . .)

Divisibility

True or false? If true, prove. If false, find a counterexample.

1. If $a|b$ and $b|c$ then $a|c$.

2. If $a|b$ and $a|c$ then $a|b$.

3. If $a|b$ and $b|a$ then $a = b$.

4. If $a|c$ and $b|c$ then either $a|b$ or $b|a$.

5. If $a|b$ and $a|c$ then $a|(mb + nc)$ for any $m, n \in \mathbb{Z}$.

6. $a|a$ for any a.

7. $a|0$ for any a.

8. $1|a$ for any a.

9. $0|a$ for any a.

10. $0|a$ if and only if $a = 0$.

11. Suppose $a|b$. Then $a|(b + c)$ if and only if $a|c$.

12. If $2|n$ and $4|n$ then $8|n$.

13. If $2|n$ and $3|n$ then $6|n$.

14. How can you tell when a number is divisible by $\{2, 3, 4, 5, 6, 7, 8, 9, 11, 17\}$?