# Chapters 2.4, 3.2

Tuesday, June 30

## Sum and Product Notation

Evaluate:

1. $\sum_{i=1}^{4} i^2 + 3i$

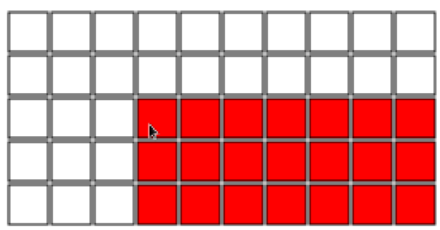
2. $\sum_{j=15}^{30} j^2$

3. $\sum_{i=1}^{3}\sum_{j=1}^{3} i + j + 1$

4. $\prod_{k=1}^{3}\sum_{i=1}^{4} ik$

Show using the sum formulas that the sum of the first $n$ positive odd numbers is equal to $n^2$.

How many rectangles of any size and orientation can be found on this $5 \times 10$ grid? (Note that perfect squares count as rectangles.)



How many rectangles on an $m \times n$ grid?

Find a formula (using summation notation) for the sum of the numbers on each row of this triangle. Bonus: show that the sum of each row is a cube.

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 3 & & 5 & & \\
 & 7 & & 9 & & 11 & \\
13 & & 15 & & 17 & & 19 \\
 & \vdots & & \vdots & & \vdots &
\end{array}$$

## Big-Oh Notation

Show (without finding an exact formula for the sum) that $\sum_{i=1}^{n} i^5$ is $O(n^6)$.

Define $a_n \sim b_n$ if $\lim_{n\to\infty} a_n/b_n = 1$. Prove: if $a_n \sim b_n$ and $b_n \sim c_n$ then $a_n \sim c_n$.

Suppose the sequence $a_n$ is $\Theta(n^k)$. If we define $b_n = \sum_{i=1}^{n} a_i$, then prove that $b_n$ is $\Theta(n^{k+1})$. What does this say about building 2-dimensional and 3-dimensional objects?

Prove that if $a_n = O(b_n)$ and $c_n = O(b_n)$ then $a_n + c_n = O(b_n)$.

Disprove: if $a_n = O(b_n)$ and $a_n = O(c_n)$ then $a_n = O(b_n + c_n)$ (think negative numbers.)

Let $a_n, b_n > 0$. Prove: $a_n = \Theta(b_n) \Leftrightarrow \ln(a_n) = \ln(b_n) + O(1)$. (What does $O(1)$ mean?)