Chapters 2.1-2.2: Sets

Friday, June 26

Set Building and Operations

1. Let the universe U be the integers. A be the set of all prime numbers. Let B be the set of even numbers. Let C be the set of all numbers greater than 10. Find an element in each of the following:

   - $A \cap B$
   - $B \cap C$
   - $A \cap C$
   - $A \cap \overline{C}$

2. Continuing with the above problem, how many elements are in A? $A \cap B$? $A \cap B \cap C$?

3. If U is the universal set, what are $\overline{\overline{U}}$ and $\overline{\overline{\emptyset}}$?

4. Find the elements of the set $[a, b] - (a, b)$.

5. Let $A = \{2z + 1 | z \in \mathbb{Z}\}$. Let $B = \{3z | z \in \mathbb{Z}\}$. Describe the set $A \cap B$ in words.

6. Express the following sets using $\mathbb{N}, \mathbb{Z}, \mathbb{R}$, and set builder notation:

   (a) The set of all even positive numbers.

   (b) The set of all non-zero real numbers.

   (c) $\{1, 2, 4, 8, 16, \ldots\}$

   (d) The set of all real numbers that are not integers.

   (e) The set of all ordered pairs of integers whose sum is 10.

7. Let $U = \{1, 2, 3, 4\}$ (the universe), $A = \{2, 3\}$, $B = \{1, 2\}$. Express each of the following sets in terms of A, B and set operations.

   (a) $\{2\}$

   (b) $\{1, 2, 3\}$

   (c) $\{1, 4\}$

   (d) $\{4\}$

   (e) $\{1, 2, 4\}$

8. Let A, B and C be three generic sets. Draw Venn diagrams for each of the following:

   (a) $A \cap (B - C)$

   (b) $A \cap (\overline{B} \cup \overline{C})$

   (c) $A - (B \cup C)$

   (d) $A \cup \overline{B} - \overline{C}$

9. What can you say about the sets A and B if

   (a) $A \cup B = A$?
   (b) $A - B = A$?
   (c) $A - B = B - A$?
   (d) $A \cap B = A$?
   (e) $A \cup B = B \cup A$?

Relation to Propositional Logic

10. What rule in propositional logic does the identity $A \cap U = A$ correspond to? What does U correspond to?

11. What does $\emptyset$ correspond to?

Proofs

Prove the following:

12. $(-\infty, a] \subset (-\infty, b]$ if and only if $a \leq b$.

13. $(-\infty, a] \subset (-\infty, b)$ if and only if $a < b$.

14. $A \subset B$ if and only if $A \cap B = A$.

15. $A \subset B$ if and only if $A \cup B = B$.

16. If $A \subset B$ and $B \subset C$ then $A \subset C$. What rule of inference does this correspond to?

17. If $A \subset B$ and $\overline{A} \subset C$ then $B \cup C = U$. What rule of inference does this correspond to?