

## Chapters 1.3-1.6: Rules of Inference

Tuesday, June 23

1. Show that the following equivalences with truth tables:

(a)  $\mathbf{T} \vee p \equiv \mathbf{T}$

(c)  $(p \vee \neg p) \equiv \mathbf{T}$

(e)  $p \Rightarrow \mathbf{T} \equiv ???$

(b)  $\mathbf{F} \wedge p \equiv \mathbf{F}$

(d)  $\mathbf{F} \vee p \equiv ???$

(f)  $p \Rightarrow \mathbf{F} \equiv ???$

2. A traveler comes upon a snake, who says this: “If you move, I will strike. If you do not move, I will strike.” Write this as a single compound proposition and show that it is logically equivalent to “I will strike.” Which rule of inference applies here?

3. Show that  $((p \Rightarrow q) \text{ and } (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$  is a tautology by at least two different methods.

4. Show that  $p \Rightarrow (q \Rightarrow r)$ ,  $q \Rightarrow (p \Rightarrow r)$ , and  $(p \wedge q) \Rightarrow r$  are all equivalent.

Back on the island of Knights and Knaves: recall that knights are virtuous and always tell the truth while knaves are wicked and always lie. There are also werewolves, who may be either knights or knaves.

You meet three people, A, B, and C. You are looking for a traveling partner and would rather go with a knight than a knave, but above all you want to avoid going with a werewolf (for obvious reasons). You happen to know that exactly one of the three islanders is a werewolf, but don't know which. Based on their statements, who should you pick as a companion?

5. A: Exactly one of us is a knave.  
B: Exactly two of us are knaves.  
C: All three of us are knaves.
6. A: At least one of us is a knight.  
B: At least one of us is a knave.  
C: The werewolf is a knight.
7. A: I am not the werewolf.  
B: The werewolf is a knave.  
C: All three of us are knaves.  
A: C is a knave!

8. Two students are wearing hats. Each can see the other's hat but not their own. The instructor informs them that each of them is wearing either a red hat or a white hat, and that at least one of them is wearing a red hat. The instructor then asks if either of the students knows the color of their own hat.

Student 1: No, I don't know.

Student 2: Well, I didn't know at first, but now I do!

What are the colors of the students' hats? How did Student 2 know?

9. Another hat problem: this time there are three students with hats. They are sitting in a straight line so that the student in back can see the other two, the middle student can see the front student, and the front student can't see anything. They are told that they are each wearing a white or a red hat, and that at least one of them is wearing a red hat.

The back student says "I do not know the color of my hat."

Next, the middle student says "I do not know the color of my hat."

The front student then says "I know the color of my hat!"

What color hats are the students wearing, and how does the front student know?