

# Chapter 1.1-1.2: Propositional Logic

Monday, June 22

1. Let  $p$ ,  $q$ , and  $r$  be the propositions “Bears have been seen in the area,” “hiking is safe on the trail,” and “berries are ripe along the trail,” respectively. Write these propositions using  $p, q, r$  and logical connectives:

- (a) Berries are ripe along the trail, but bears have not been seen in the area.

$$r \wedge \neg p$$

- (b) If berries are ripe along the trail, hiking is safe if and only if bears have not been seen in the area.

$$r \Rightarrow (q \Leftrightarrow \neg p)$$

- (c) Hiking is not safe on the trail whenever bears have been seen in the areas and berries are ripe along the trail.

$$(p \wedge r) \Rightarrow q$$

2. Rewrite the following phrases in the form “If P, then Q.” State the contrapositive, inverse, and converse. Decide whether the biconditional “P if and only if Q” is implied.

- (a) In order to run for president, you must be at least 35.

- i. If you can run for president then you are at least 35.
- ii. Inverse: If you cannot run for president then you are under 35.
- iii. Converse: If you are at least 35 then you can run for president.
- iv. Contrapositive: If you are not at least 35 then you cannot run for president.

- (b) You can't make an omelet without breaking eggs.

- i. If you made an omelet, then you broke eggs.
- ii. Inverse: If you did not make an omelet then you did not break eggs.
- iii. Converse: If you break eggs then you can make an omelet.
- iv. Contrapositive: If you do not break eggs then you cannot make an omelet.

- (c) Surrender or die!

- i. If you do not surrender, you will die.
- ii. Inverse: If you surrender, you will not die.
- iii. Converse: If you die, you did not surrender.
- iv. Contrapositive: If you do not die, then you surrendered.

- (d) Be careful or you might hurt yourself.

- i. If you are not careful you (will) hurt yourself.
- ii. Inverse: If you are careful you will not hurt yourself.
- iii. Converse: If you hurt yourself then you were not careful.
- iv. Contrapositive: If you did not hurt yourself then you were careful.

- (e) No shirt, no shoes, no service.

- i. If you are not wearing a shirt or you are not wearing shoes, then you will not get service.
- ii. Inverse: If you are wearing a shirt and shoes, then you will get service.
- iii. Converse: If you did not get service then you were not wearing a shirt or were not wearing shoes.
- iv. Contrapositive: If you got service then you were wearing a shirt and shoes.

- (f) Everything will be alright if we just keep dancing like we're 22.
- If we dance then everything will be alright.
  - Inverse: If you do not dance then not everything will be alright.
  - Converse: If everything is alright then we were dancing.
  - Contrapositive: If everthing is not alright then we were not dancing.
- (g) A watched pot never boils.
- If you watch the pot then it will not boil.
  - Inverse: If you do not watch the pot then it will boil.
  - Converse: If the pot does not boil then you were watching it.
  - Contrapositive: If the pot boils then you were not watching it.
- (h) You can't order alcohol unless you're over 21.
- If you can order alcohol then you are over 21.
  - Inverse: If you cannot order alcohol then you are under 21.
  - Converse: If you are over 21 then you can order alcohol.
  - Contrapositive: If you are under 21 then you cannot order alcohol.
- (i) The Warriors will win as long as they play well.
- If the Warriors play well then they will win.
  - Inverse: If the Warriors do not play well then they will not win.
  - Converse: If the Warriors won then they played well.
  - Contrapositive: If the warriors did not win then they did not play well.
- (j) The Cubs lose whenever I watch them play.
- If I watch the Cubs then they lose.
  - Inverse: If I do not watch the Cubs then they win.
  - Converse: If the Cubs lose then I was watching them.
  - Contrapositive: If the Cubs win then I was not watching them.
3. There is a restaurant in Chicago that displays the following sign:

No shirt  
No shoes  
No pets  
No bikes  
No service

Please explain.

If you are not wearing a shirt or not wearing shoes, or if you have a bike or a pet, then you will not get service. It really doesn't make a lot of sense the way it's written.

4. Construct truth tables for each of the following:

- (a) not (P and Q)

$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$\neg(p \wedge q)$
$T$	$T$	$F$	$F$	$F$	$T$	$F$
$T$	$F$	$F$	$T$	$T$	$F$	$T$
$F$	$T$	$T$	$F$	$T$	$F$	$T$
$F$	$F$	$T$	$T$	$T$	$F$	$T$

(b) (not P) or (not Q)

*See above. It's the same as  $\neg(p \wedge q)$*

(c)  $(P \text{ or } Q) \Rightarrow P$

$p$	$q$	$p \vee q$	$(p \vee q) \Rightarrow p$
$T$	$T$	$T$	$T$
$T$	$F$	$T$	$T$
$F$	$T$	$T$	$F$
$F$	$F$	$F$	$T$

5. Consider the statement “All girls are good at math.” Which of the following statements mean the same thing as this statement? Which ones mean the same thing as its negation?
- (a) All girls are bad at mathematics.
  - (b) All girls are not good at mathematics.
  - (c) Some girl is bad at mathematics. NEGATION, assuming “not good” is the same as “bad.”
  - (d) Some girl is not good at mathematics. NEGATION
  - (e) All children who are good at mathematics are girls.
  - (f) All children who are not good at mathematics are boys. SAME
6. Which of the following are necessary conditions for a number  $n$  to be divisible by 6? Which are sufficient conditions? Reword your responses as if-then statements.
- (a)  $n$  is divisible by 3. Necessary but not sufficient: 3 is divisible by 3 but not 6.
  - (b)  $n$  is divisible by 9. Not necessary since 6 is not divisible by 9. Not sufficient since 9 is not divisible by 6.
  - (c)  $n$  is divisible by 12. Sufficient but not necessary: 18 is divisible by 6 but not 12.
  - (d)  $n^2$  is divisible by 6. Necessary and sufficient (but not obviously so).
  - (e)  $n = 12$ . Sufficient but not necessary.
  - (f)  $n$  is divisible by 2 and 3. Necessary and sufficient.

Welcome to the island of Knights and Knaves! Knights are virtuous and always tell the truth. Knaves are wicked and always lie. Suppose you meet two people, A and B. What can you tell about them from their statements?

7. A says “I am a knight” and B also says “I am a knight.”  
You can’t tell anything—they could be telling the truth or lying.
8. A says “We are both knights” and B says “A is lying!”  
The statements contradict each other, so at least one of them is lying. “A” claims that they are both knights, so A is lying. B is therefore telling the truth.  
Conclusion: A is a knave and B is a knight.
9. A says “We are both knaves” and B says nothing.  
If “A” were a knight, his statement would imply that he is a knave, which is a contradiction. “A” must therefore be a knave.  
Since “A” claims that both A and B are knaves, but A is lying, at least one of the two is a knight. Since A is a knave, B must be a knight.
10. A says “B is a knight” and B says “A is a knight.”  
They are either both knights or both knaves: if one were a knight and the other were a knave, the knight would have said so.
11. A says “Either I am a knave or B is a knight” and B says nothing.  
If A were a knave, then his statement that he was a knave would be true, a contradiction. Thus A is a knight. Since A is telling the truth he cannot be a knave, and so for his statement to be true B must also be a knight.