Math 55: Practice Midterm 3

Midterm: Friday, July 31

1. 50 people go out to eat. Everyone orders either a hamburger or a salad. 15 people put mustard on their burgers, 25 put ketchup on their burgers, and 10 people put both ketchup and mustard on their burgers. How many people ordered a salad?

2. How many times must I roll a pair of dice in order to guarantee that I roll some number (the sum of the two dice) twice?

3. How many ways are there to put 3 red chairs and 4 blue chairs around a circular table if chairs of the same color are indistinguishable and two arrangements that differ only by rotating the table count as the same?

4. How many distinct ways are there to put 2 red dots and 4 blue dots on the faces of a (blank and symmetrical) cube so that each face gets one dot?

5. Evaluate the sum $\sum_{i=0}^{20} \binom{n}{i} (-1)^i 2^{n-i}$

6. I have 3 teal balls, 4 magenta balls, and 5 orange balls in a cauldron. If I draw 3 balls without replacement, what is the probability that I get 2 orange balls and 1 magenta? What if I draw 3 balls with replacement?

7. How many ways are there to give 8 cookies to 4 friends if every friend must get at least 1 cookie?

8. How many ways are there to buy 7 fruit if my options are apples, bananas, and peaches?

9. How many ways are there to give 5 blue hats, 2 red hats, and 3 green hats to 10 friends?

10. If I am dealt a random hand of 5 cards, what is the probability of getting a straight (e.g. 2-3-4-5-6 or 8-9-10-J-Q in any combination of suits; A-2-3-4-5 is not okay but 10-J-Q-K-A is fine).

11. There is a 50% chance that it rains tomorrow and a 30% chance that I will go outside. If these are independent events, what is the chance that it rains but I stay inside?

12. A fair coin and a loaded coin (p(heads) = .7) are sitting on a table. If I take a random coin and flip 6 heads out of 10, what is the chance that I took the fair coin?

13. I roll two dice. If X is the sum of the rolls and Y is the product of the rolls, prove that X and Y are not independent random variables.

14. I flip a coin five times and get 2^n dollars for every time I flip heads. What is the expected amount of money I will make in the game?

15. I roll a die, multiply the result by 3, then add 7. What is the expected value of my final number? What is the variance?

16. Prove that if E and F are independent events then $\overline{E}$ and F are also independent events.

17. Prove that if E and F are independent random variables and $G = 2E+3$ then G and F are independent random variables.