Math 55: Practice Midterm 2

Midterm: Friday, July 17

1. Find $\sum_{i=1}^{2} \prod_{j=1}^{3} (i+j)$
2. Define $a_1 = 1$ and for $n \geq 2$ define $a_n = \sum_{i=1}^{n-1} a_i$.
   (a) Find a_5.
   (b) Find a formula for a_n where $n \geq 2$ and prove that it is correct.
3. Find the greatest common divisor of 184 and 306.
4. Find a solution to $184x + 306y = d$, where $d = \gcd(184, 306)$.
5. Solve: $184x \equiv 16 \pmod{306}$.
6. Find all solutions to $36x \equiv 17 \pmod{60}$.
7. Find all solutions to $36x \equiv 18 \pmod{60}$.
8. Evaluate the following:
   (a) $815 \pmod 7$
   (b) $23234 \cdot 101 \pmod 4$
   (c) $(-17) \cdot 82 \pmod 3$
   (d) $5^{88} \pmod 6$
   (e) $98 \cdot 96 \pmod{99}$
   (f) $2^{87} \pmod 7$
   (g) $2^{87} \pmod{35}$
9. Prove that 101 is prime.
10. Say whether each of the following equations has an integer solution:
    (a) $x \equiv 18 \pmod{45}, x \equiv 1 \pmod 2$
    (b) $x \equiv 13 \pmod{15}, x \equiv 5 \pmod 6$
    (c) $x \equiv 12 \pmod{15}, x \equiv 3 \pmod 6$
11. Prove that if $a \equiv b \pmod m$ then $-a \equiv m - b \pmod m$.
12. Prove that the equation $x^2 + 3x + 5y = 1$ has no solutions where x and y are integers.
13. How many divisors does 100 have?
14. How many numbers are relatively prime to 100?
15. How many zeroes does 100! end with?