Math 55: Practice Midterm 1
Midterm: Thursday, July 2

1. Prove using truth tables that $\neg(p \wedge q) \wedge p$ is equivalent to $\neg q \wedge p$.

2. Let w be the statement "It is Wednesday," d be "I have a dollar," and s be "I can buy a shake." Write the following using w, d, s, and logical connectives:

   (a) I can buy a shake if today is Wednesday and I have a dollar.

   (b) Today is Wednesday, but I do not have a dollar.

   (c) I need a dollar in order to buy a shake.

3. Assume the previous three statements are all true. Prove the following:

   (a) If I cannot buy a shake but I have a dollar, then today is not Wednesday.

   (b) If today is Wednesday, then I can buy a shake if and only if I have a dollar.

4. Draw a Venn Diagram showing the relation between $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$, and S the set of all numbers divisible by 3.

5. What is the contrapositive of "If today is Tuesday or Wednesday then pizza is on sale"? What about the inverse and converse?

6. Let $A = \{1, 2, 3\}$, $B = \{2, 4\}$, and let the universe U be $\{1, 2, 3, 4\}$. Describe the following using only A and B and set operations:

   (a) $\{1, 3\}$

   (b) $\{2\}$

   (c) $\{1, 3, 4\}$

   (d) $\{4\}$

7. Draw a Venn Diagram, and prove: if $A \subset B$ then $\overline{B} \subset \overline{A}$.

8. Prove that $3a + 2$ is even if and only if a is even for integers a.

9. Prove that for any integer n, $n(n^2 - 1)(n^2 + 1)$ is divisible by 5. (You do not need to expand the entire expression! Just show that at least one of the three factors is divisible by 5).

10. Express in quantifier notation, and prove or disprove:

    (a) There is no smallest positive real number a.

    (b) The interval $[0, 2]$ has a largest element.

    (c) The square of any real number is positive.