1. Show that the expression \((p \Rightarrow q) \Rightarrow (q \Rightarrow p)\) is neither a tautology nor a contradiction.

2. State the negation and prove or disprove: \((\forall x)(\exists y)(\forall z)(xy \geq z)\)

3. Prove that if \(x\) and \(y\) are positive then \(\sqrt{\frac{x^2 + y^2}{2}} \geq \frac{x+y}{2}\).

4. Evaluate: \(\sum_{i=1}^{10} \sum_{j=1}^{i} i - 2j\)

5. Find integers \(x, y \in \mathbb{Z}\) such that \(18x + 40y = 14\)

6. Determine whether each of the systems of equations has a solution:
   
   (a) \[
   \begin{align*}
   x &\equiv 15 \pmod{35} \\
   x &\equiv 8 \pmod{10} \\
   x &\equiv 1 \pmod{7}
   \end{align*}
   \]
   
   (b) \[
   \begin{align*}
   x &\equiv 3 \pmod{6} \\
   x &\equiv 7 \pmod{8} \\
   x &\equiv 4 \pmod{5}
   \end{align*}
   \]

7. Prove using induction that if \(G\) is a tree with at least 2 vertices then \(\chi(G) = 2\). You may use the fact that every tree with 2 or more vertices has at least 2 vertices of degree 1.

8. State the inverse, converse, and contrapositive, and prove or disprove each one: “If a number is divisible by 4 and 5 then it is divisible by 20.”

9. I draw cards from a deck until I have drawn all 4 aces. What is the expected number of kings that I will have drawn?

10. Prove that if the events \(E\) and \(F\) are positively correlated then the events \(E\) and \(\overline{F}\) are negatively correlated.

11. 10 cows, 10 ducks, and 10 pigs are all standing in a line, their positions distributed at random. What is the expected number of times a cow will be standing directly in front of a duck?

12. If I flip a fair coin 40 times, prove that the probability of getting 30 or more heads is less than or equal to \(1/20\).

13. There is an urn with 5 red balls and 3 yellow balls. I draw 2 balls from the urn, flipping a fair coin to decide whether to draw with or without replacement. If I draw 1 red ball and 1 yellow, what is the probability that I drew without replacement?

14. Give an example of each of the following:
   
   (a) A connected graph with no cycles.
   
   (b) A graph where every vertex has degree 3.
(c) A graph with an Euler path but no Euler circuit.
(d) A graph with a Hamilton cycle but no Euler path.
(e) A graph with $\chi(G) = \alpha(G) = \omega(G) = 4$.
(f) A non-planar triangle-free graph.

15. Remove an edge of your choice from $K_5$. How many automorphisms does the resulting graph have?

16. I glue triangles and squares together in the shape of a ball so that 4 shapes fit together at every vertex. Show that the number of triangles needed is the same no matter how many squares are used.