Practice Final

Final: Friday, August 14

NAME: ____________________________

1. Show that the expression $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$ is neither a tautology nor a contradiction.
2. State the negation and prove or disprove: $(\forall x)(\exists y)(\forall z)(xy \geq z)$
3. Prove that if x and y are positive then $\sqrt{\frac{x^2+y^2}{2}} \geq \frac{x+y}{2}$.
4. Evaluate: $\sum_{i=1}^{10} \sum_{j=1}^{i} i - 2j$
5. Find integers $x, y \in \mathbb{Z}$ such that $18x + 40y = 14$
6. Determine whether each of the systems of equations has a solution:

   (a)

$$\begin{aligned} x &\equiv 15 \pmod{35} \\ x &\equiv 8 \pmod{10} \\ x &\equiv 1 \pmod{7} \end{aligned}$$

   (b)

$$\begin{aligned} x &\equiv 3 \pmod{6} \\ x &\equiv 7 \pmod{8} \\ x &\equiv 4 \pmod{5} \end{aligned}$$

7. Prove using induction that if G is a tree with at least 2 vertices then $\chi(G) = 2$. You may use the fact that every tree with 2 or more vertices has at least 2 vertices of degree 1.
8. State the inverse, converse, and contrapositve, and prove or disprove each one: "If a number is divisible by 4 and 5 then it is divisible by 20."
9. I draw cards from a deck until I have drawn all 4 aces. What is the expected number of kings that I will have drawn?
10. Prove that if the events E and F are positively correlated then the events E and $\overline{F}$ are negatively correlated.
11. 10 cows, 10 ducks, and 10 pigs are all standing in a line, their positions distributed at random. What is the expected number of times a cow will be standing directly in front of a duck?
12. If I flip a fair coin 40 times, prove that the probability of getting 30 or more heads is less than or equal to 1/20.
13. There is an urn with 5 red balls and 3 yellow balls. I draw 2 balls from the urn, flipping a fair coin to decide whether to draw with or without replacement. If I draw 1 red ball and 1 yellow, what is the probability that I drew without replacement?
14. Give an example of each of the following:
    (a) A connected graph with no cycles.
    (b) A graph where every vertex has degree 3.

(c) A graph with an Euler path but no Euler circuit.

(d) A graph with a Hamilton cycle but no Euler path.

(e) A graph with $\chi(G) = \alpha(G) = \omega(G) = 4$.

(f) A non-planar triangle-free graph.

15. Remove an edge of your choice from K_5. How many automorphisms does the resulting graph have?

16. I glue triangles and squares together in the shape of a ball so that 4 shapes fit together at every vertex. Show that the number of triangles needed is the same no matter how many squares are used.