

# Math 55: Midterm 3

Friday, July 31

NAME: \_\_\_\_\_

1. (2 points each) Evaluate:

(a)  $\binom{6}{3}$

(b)  $\binom{10}{10}$

(c)  $\binom{8}{9}$

(d)  $\binom{180}{179}$

2. (5 points) Four juniors and six seniors form a club. How many ways are there to pick a president and a vice president if the president must be a senior but the vice president can be either a junior or a senior? (The two offices must go to different people.)

3. (6 points) How many integers  $n$  with  $1 \leq n \leq 1000$  are **not** divisible by 4 and **not** divisible by 5?

4. (5 points) I flip a coin. If it lands on tails, I get nothing. If it lands on heads, I roll a die and collect  $n$  dollars for rolling the number  $n$ . What is the expected value of the amount of money I will make?

5. (5 points) I have 10 identical coins to give to Alice, Bob, Carol, and Dwight. How many different ways can I distribute the coins if I give Bob at least 2 coins? You may leave your answer in the form  $\binom{n}{k}$ .

6. (5 points) I have a coin that lands on heads  $2/3$  of the time and tails  $1/3$  of the time. If I flip the coin 4 times, what is the probability that I get 2 heads? Write your answer as a single fraction  $p/q$ .

7. (5 points) I have two urns. Urn A contains 5 red balls and 3 green balls. Urn B contains 1 red ball and 1 green ball. I pick an urn at random and draw a random ball from it. If I draw a red ball, what is the probability that I picked Urn A? Write your answer as a single fraction  $p/q$ .

8. (6 points) Prove that if  $E$  and  $\overline{E}$  are independent events then  $p(E) = 0$  or  $p(E) = 1$ .

9. (5 points) There is a 30% chance overall that I will have a picnic tomorrow. There is a 40% chance of rain tomorrow. If it rains then there is a 0% chance that I will have a picnic. What is the chance that I will have a picnic if it does not rain?