

# Math 55: Midterm 3

Friday, July 31

NAME: \_\_\_\_\_

1. (2 points each) Evaluate:

(a)  $\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

(b)  $\binom{10}{10} = 1$

(c)  $\binom{8}{9} = 0$

(d)  $\binom{180}{179} = 180$

2. (5 points) Four juniors and six seniors form a club. How many ways are there to pick a president and a vice president if the president must be a senior but the vice president can be either a junior or a senior? (The two offices must go to different people.)

There are 6 ways to pick a senior for president and then  $(4+6-1) = 9$  ways to pick a vice president, so  $6 \cdot 9 = 54$  ways in all.

3. (6 points) How many integers  $n$  with  $1 \leq n \leq 1000$  are **not** divisible by 4 and **not** divisible by 5?

250 are divisible by 4, 200 by 5, and 50 by 4 and 5, so  $200+250-50 = 400$  are divisible by 4 or 5. Therefore 600 are divisible by neither.

4. (5 points) I flip a coin. If it lands on tails, I get nothing. If it lands on heads, I roll a die and collect  $n$  dollars for rolling the number  $n$ . What is the expected value of the amount of money I will make?

Half chance of nothing, half chance of expected value 3.5, so total expected value  $3.5/2 = 1.75 = 7/4$ .

5. (5 points) I have 10 identical coins to give to Alice, Bob, Carol, and Dwight. How many different ways can I distribute the coins if I give Bob at least 2 coins? You may leave your answer in the form  $\binom{n}{k}$ .

Give Bob 2 coins, then stars and bars with 8 coins and 4 people, so  $\binom{8+4-1}{4-1} = \binom{11}{3}$ .

6. (5 points) I have a coin that lands on heads  $2/3$  of the time and tails  $1/3$  of the time. If I flip the coin 4 times, what is the probability that I get 2 heads? Write your answer as a single fraction  $p/q$ .

$$\binom{4}{2} (1/3)^2 (2/3)^2 = 24/81.$$

7. (5 points) I have two urns. Urn A contains 5 red balls and 3 green balls. Urn B contains 1 red ball and 1 green ball. I pick an urn at random and draw a random ball from it. If I draw a red ball, what is the probability that I picked Urn A? Write your answer as a single fraction  $p/q$ .

$$p(A|r) = \frac{p(r|A)p(A)}{p(r|A)p(A)+p(r|B)p(B)} = \frac{(5/8)(1/2)}{(5/8)(1/2)+(1/2)(1/2)} = \frac{5}{9}.$$

8. (6 points) Prove that if  $E$  and  $\bar{E}$  are independent events then  $p(E) = 0$  or  $p(E) = 1$ .

$$0 = p(E \cap \bar{E}) = p(E)p(\bar{E}), \text{ so } p(E) = 0 \text{ or } p(\bar{E}) = 0 \text{ (in which case } p(E) = 1 - 0 = 1).$$

9. (5 points) There is a 30% chance overall that I will have a picnic tomorrow. There is a 40% chance of rain tomorrow. If it rains then there is a 0% chance that I will have a picnic. What is the chance that I will have a picnic if it does not rain?

$$p(P|\bar{r}) = \frac{p(P \cap \bar{r})}{p(\bar{r})} = \frac{p(P)}{p(\bar{r})} = \frac{.3}{.6} = .5.$$