Math 55: Midterm 2

Friday, July 17

NAME: ____________________________

1. (2 points each) Evaluate:

   (a) $\prod_{i=1}^{3}\sum_{j=1}^{2}(j+1)$

   (b) $\sum_{\substack{d\geq 0\\ d|15}} d$

   (c) $\sum_{1\leq i\leq j\leq 3} j$

2. (1 point each) Compute:

   (a) $157\cdot 15 \mod 7$

   (b) $369\cdot 377 \mod 373$

   (c) $2^{364} \mod 7$

   (d) $38^{38} \mod 3$

3. True or False: $(\exists x\in\mathbb{Z})(\forall y,z\in\mathbb{Z})(13y+40z\neq x)$. (1 point for answer, 2 for explanation)

4. (3, 3, and 2 points)

   (a) Use the Euclidean Algorithm to find the greatest common divisor of 120 and 35.

   (b) Find any two integers x and y so that $120x + 35y = \gcd(120, 35)$.

   (c) Find the least common multiple of 120 and 35.

5. (5 points) Prove that the following system of congruences has no integer solution:

   $x \equiv 5 \pmod{30}$
   $x \equiv 11 \pmod{12}$
   $x \equiv 7 \pmod{15}$

6. Define the Fibonacci sequence by $f_0 = 0$, $f_1 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 1$.
Define the Lucas sequence by $l_0 = 2, l_1 = 1$, and $l_{n+1} = l_n + l_{n-1}$ for $n \geq 1$.

   (a) (2 points) Find l_6.

   (b) (6 points) Prove that $l_n + l_{n+2} = 5f_{n+1}$ for all $n \geq 0$.

7. (6 points) Prove that if $a|m$ and $b|n$ then $ab|mn$.

8. (4 points) Find all solutions to $x^2 + 4x \equiv 15 \pmod{19}$ with $0 \leq x < 19$.

9. (3 points) Find all solutions to $x^2 \equiv 35 \pmod{65}$ with $0 \leq x < 65$.

10. (3 points) Let $a = 5k + 8$ and let $b = 4k + 3$ for some integer k. Show that $\gcd(a, b)$ is either 1 or 17.