

# Math 55: Homework 9

Due Friday, July 24

1. Prove the *hockeystick identity*

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}$$

when  $n, r \geq 0$  by

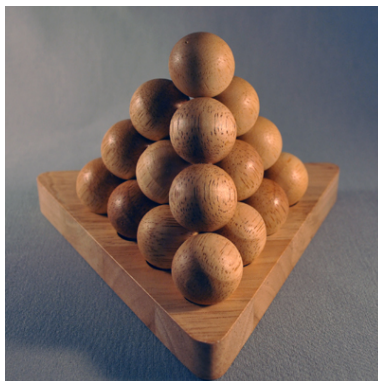
- (a) using a combinatorial argument. (You want to choose  $r$  objects. For each  $k$ : choose the first  $r-k$  in a row, skip one, then how many choices do you have for the remaining objects?)

- (b) Using Pascal's identity (plus induction! For the induction, fix  $n$  arbitrarily and then use induction on  $r$ )

2. How many ways can you split nine people into three groups of three?

3. How many ways can you give nine (identical) cookies to three friends? It is okay to give no cookies to some of those friends.

4. Remember that problem with the spheres and the triangular pyramid from an earlier homework? Here it is again!



- (a) Use the hockeystick identity to show that  $T_n = 1 + 2 + \dots + n$  is equal to  $\binom{n+1}{2}$ .

- (b) Then use the hockeystick identity to show that  $T_1 + T_2 + \dots + T_n = \binom{n+2}{3}$ .

- (c) Give an illustration of the above using Pascal's triangle.