Math 55: Homework 7

Due Wednesday, July 13

1. Define a pair of sequences as follows: $a_0 = 1, b_0 = 1$, and $a_{n+1} = a_n + 2b_n, b_{n+1} = a_n + b_n$ for $n \geq 0$.

   (a) If we define $c_n = a_n/b_n$, write out $c_0, c_1, c_2, \ldots c_6$.

   (b) Prove by induction that $2b_n^2 - a_n^2 = (-1)^n$.

   (c) (OPTIONAL) What is $\lim_{n\to\infty} c_n$?

2. For integers $n \geq 1$, define $S(n)$ as follows:

$$S(n) = \begin{cases} n/2 & \text{if n is even} \\ (n+1)/2 & \text{if n is odd} \end{cases}$$

   Using strong induction, prove that no matter what number n we begin at the sequence

$$n, S(n), S(S(n)), S(S(S(n))), \ldots$$

   will eventually reach the number 1. (For example, if $n = 11$ we get the sequence $11, 6, 3, 2, 1$.)