Math 55: Homework 5
Due Thursday, July 9

1. Express the following using sum and product notation:

\[ 1 + (2 \cdot 3) + (3 \cdot 4 \cdot 5) + (4 \cdot 5 \cdot 6 \cdot 7) + \ldots + (8 \cdot 9 \cdot 10 \cdot 11 \cdot 12 \cdot 13 \cdot 14 \cdot 15) \]

\[ \sum_{i=1}^{8} \prod_{j=0}^{i-1} (i + j) \]

2. Show that a number \( n \) is divisible by 4 if and only if one of these two cases holds: (1) the tens digit is even and the ones digit is divisible by 4, or (2) the tens digit is odd and the ones digit is equivalent to 2 (mod 4).

\[ 4 | 100h + 10t + u \iff 4 | 10t + u \]
\[ \iff 4 | 2t + u \]

\( 2t + u \) is divisible by 4 only when \( u \) is even, let \( u = 2k \). This means we need \( 2t + 2k = 4n \), so our number is divisible by 4 if and only if \( t + k \) is even. This is true if and only if \( t \) and \( k \) are both even (meaning \( t \) is even and \( u \) is divisible by 4) or \( t \) and \( k \) are both odd (meaning \( t \) is odd and \( u = 2(2s + 1) = 4s + 2 \equiv 2 \) (mod 4)).

Or, once you get the condition \( 4 | 2t + u \), make a table of \( t \) and \( u \) and \( 2t + u \) mod 4:

<table>
<thead>
<tr>
<th>t, u</th>
<th>0 1 2 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 1 2 3</td>
</tr>
<tr>
<td>1</td>
<td>2 3 0 1</td>
</tr>
<tr>
<td>2</td>
<td>0 1 2 3</td>
</tr>
<tr>
<td>3</td>
<td>2 3 0 1</td>
</tr>
</tbody>
</table>

The only cases when \( 2t + u \equiv 0 \) (mod 4) are when \( t \) is even and \( u \) is 0 mod 4 or when \( t \) is odd and \( u \) is 2 mod 4.

3. Show that 2034956098435602302 is not a perfect square. Your proof should not involve multiplying any large numbers.

The last digit is 2, but \( n^2 \) is never 2 mod 10.

4. What are the last two digits of \( 341899^{100} \)?

“Last two digits” is the same as the remainder mod 100, so \( 341899^{100} \pmod{100} = (-1)^{100} \pmod{100} = 1 \). The last two digits are “01.”