

Math 55: Homework 4

Due Tuesday, July 7

1. Here is a picture of a triangular pyramid made of spheres:



- How many spheres are in the pyramid?
- Using double sums, write a formula for the number of spheres in a pyramid with n layers.
- Find a closed form for the double sum in your previous answer. How many spheres are in a pyramid with 50 layers?
- If a_n is the number of spheres in a pyramid with n layers, what is the order of growth of $\{a_n\}$?
(e.g $\Theta(n)$, $\Theta(n^2)$, etc.)

2. Here is a multiplication table for numbers up to 12:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

(a) Use summation formulas to find the sum of all numbers in the table (excluding the numbers in bold in the outer row and column).

(b) What is the sum of the numbers in an $n \times n$ table?

3. Prove or find a counterexample:

(a) If $\{a_n\} = O(b_n)$ and $\{b_n\} = O(c_n)$ then $a_n = O(c_n)$.

(b) If $\{a_n\}$ and $\{b_n\}$ are both $\Theta(c_n)$, then $\{a_n + b_n\}$ is also $\Theta(c_n)$.