

# Math 55: Homework 3

Due Tuesday, June 30

1. Define the universal set as  $U = \{1, 2, 3, 4\}$ . Let  $A = \{1, 2\}$ ,  $B = \{2, 3\}$ . Express the following sets in terms of  $A$ ,  $B$ , and set operations:
  - (a)  $\{1, 2, 3\}$
  - (b)  $\{1, 4\}$
  - (c)  $\{4\}$
  - (d)  $\{1, 3, 4\}$
  
2. Let  $A$  and  $B$  be sets in the universe  $U$ , and suppose that  $\overline{A} \subset B$ . Prove that  $A \cup B = U$ .
  
  
  
  
  
  
  
  
  
  
3. Let  $C$  be the set of all countries,  $R$  be the set of all ravens, and  $B$  be the set of all black animals. Let  $L(x, y)$  mean “animal  $x$  lives in country  $y$ .” Express the following statements in quantifier notation. Write their negations in English *and* in quantifier notation.
  - (a) All ravens are black.
  - (b) In every country there lives at least one non-black raven.
  
  
  
  
  
  
  
  
  
  
4. Write the statement in English, and prove or find a counterexample:  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$ .