1. Define the universal set as $U = \{1, 2, 3, 4\}$. Let $A = \{1, 2\}, B = \{2, 3\}$. Express the following sets in terms of $A, B$, and set operations:

   (a) $\{1, 2, 3\} = A \cup B$
   (b) $\{1, 4\} = \overline{B}$
   (c) $\{4\} = \overline{A} \cup B = \overline{A} \cap \overline{B}$
   (d) $\{1, 3, 4\} = \overline{A} \cap B = \overline{A} \cup B$

2. Let $A$ and $B$ be sets in the universe $U$, and suppose that $A \subset B$. Prove that $A \cup B = U$.

   Since $A \cup B \subset U$ by definition, we just need to prove that $U \subset A \cup B$. . . that is, for any $x$ that $x \in A \cup B$.

   Take any $x \in U$. If $x \in A$, then $x \in A \cup B$. If $x \notin A$, then $x \in \overline{A}$ by definition. Since it is a given that $A \subset B$, it follows that $x \in B$, and so $x \in A \cup B$.

   Therefore, $x \in A \cup B$ for any $x \in U$, and so $A \cup B = U$.

3. Let $C$ be the set of all countries, $R$ be the set of all ravens, and $B$ be the set of all black animals. Let $L(x, y)$ mean “animal $x$ lives in country $y.$” Express the following statements in quantifier notation. Write their negations in English and in quantifier notation.

   (a) All ravens are black.
      i. $(\forall x \in R)(x \in B)$, or $(\forall x)(x \in R \Rightarrow x \in B)$.
      ii. “Not all ravens are black,” or “There is a non-black raven.”
      iii. $(\exists x \in R)(x \notin B)$.

   (b) In every country there lives at least one non-black raven.
      i. $(\forall c \in C)(\exists x \in R)(L(x, c) \land x \notin B)$ (there may be other valid ways of writing this).
      ii. “There is a country with no non-black ravens,” or “There is a country in which all ravens are black.”
      iii. $(\exists c \in C)(\forall x \in R)(L(x, c) \Rightarrow x \in B)$. It would not be appropriate to say $(\exists c \in C)(\forall x \in R)(L(x, c) \land x \in B)$, since that would imply that there is one country in which all ravens live.

4. Write the statement in English, and prove or find a counterexample: $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$.
   “For every real number $x$ there is some real number $y$ such that $xy = 1.$”
   Counterexample: $x = 0.$