Chapter 5.?
Wednesday, Week 4

Divisors

Here are all the divisors of 18 laid out in a rectangle. We'll draw an arrow bewteen two numbers a and b if $b = a \cdot p$ for some prime p.

$$\begin{array}{ccccc} 1 & \to & 3 & \to & 9 \\ \downarrow & & \downarrow & & \downarrow \\ 2 & \to & 6 & \to & 18 \end{array}$$

Make a similar diagram for the divisors of 72.

How many powers of 3 are there in 9!? How many powers of 2 in 8!?

Euler's Phi Function

1. If $\gcd(a, c) = \gcd(b, c) = 1$ then what is $\gcd(ab, c)$?

2. How many numbers $0 \leq x < 17$ are there with $\gcd(x, 17) = 1$?

3. How many numbers $0 \leq x < 27$ with $\gcd(x, 27) = 1$?

4. How many numbers $0 \leq x < 81$ with $\gcd(x, 81) = 1$?

The Primes

What is $\gcd(a, abc + 1)$?

Take $a, b, c \geq 2$ with $\gcd(a, b) = \gcd(a, c) = \gcd(b, c) = 1$. Can a, b, and c share any prime factors?

What is $\gcd(k, n! + 1)$ if $1 \leq k \leq n$?