Chapter 5.2
Tuesday, Week 4

Warmup

13 is a fluffy number. If a natural number n is fluffy then $n+1$ is also fluffy. Which numbers must be fluffy?

10 is not a fluffy number. Which numbers cannot be fluffy?

Let a_n be a sequence such that $a_{n+1} \geq a_n$ for all n. If $a_0 = 1$, prove that $a_n \geq 1$ for all $n \in \mathbb{N}$.

The Well-Ordering Property

Let S and T be sets with $S \subset \mathbb{N}$ and $T \subset \mathbb{Z}$. Which of these statements *must* be true?

1. S has a largest element.
2. T has a largest element.
3. S has a smallest element.
4. T has a smallest element.

Find a subset of the interval $[0, 1]$ with no largest or smallest element.

Strong Induction

In a game of football you can score 6, 7, or 8 points for a touchdown and 3 points for a field goal. What scores can you get with only touchdowns and field goals?

Suppose that you can only score 6 or 7 points with a touchdown (because you never try for the 2-point conversion). What scores can you get now?

The Primes

What is $\gcd(a, abc + 1)$?

Take $a, b, c \geq 2$ with $\gcd(a, b) = \gcd(a, c) = \gcd(b, c) = 1$. Can a, b, and c share any prime factors?

What is $\gcd(k, n! + 1)$ if $1 \leq k \leq n$?