Chapter 5.1
Monday, Week 4

Warmup

How many solutions mod 100... one, many, or none?

1. \( x \equiv 7 \pmod{25}, x \equiv 3 \pmod{4} \)
2. \( x \equiv 3 \pmod{5}, x \equiv 18 \pmod{20} \)
3. \( x \equiv 3 \pmod{5}, x \equiv 16 \pmod{20} \)

True or false: \( 2^{28} \equiv 1 \pmod{29} \)

True or false: \( 2^{42} \equiv 1 \pmod{33} \)

Induction

Define: A curious number is a number that is curious. Suppose we know two things about curious numbers:

1. If any integer \( n \) is a curious number, then \( n + 2 \) is a curious number.
2. 7 is a curious number.

Which numbers must also be curious?

1. 5 4. 15 7. 39523092357
2. 9 5. 1341 8. \( n \)
3. 10 6. 2808 9. \( \infty \)

There is a machine that makes widgets all day. It has one problem— if a widget it makes is defective, then the next widget it makes will also be defective. What can you say about the machine?
Recursion

Define \( f_n = \begin{cases} 
0 & n = 0 \\
1 & n = 1 \\
& f_{n-1} + f_{n-2} & n \geq 2
\end{cases} \). Find \( f_2, f_3, f_4 \) and \( f_5 \).

Which Fibonacci numbers are even? Which ones are divisible by 3?

Because of the Euclidean Algorithm, we can compute the gcd of two non-negative integers with a recursive function: \( \text{gcd}(a, b) = \begin{cases} 
a & b = 0 \\
\text{gcd}(b, a) & b \geq a \\
\text{gcd}(a - b, b) & 0 < b < a
\end{cases} \). Use this definition to find \( \text{gcd}(13, 8) \).