Chapter 5.1
Monday, Week 4

Warmup

How many solutions mod 100. . . one, many, or none?

1. $x \equiv 7 \pmod{25}$, $x \equiv 3 \pmod{4}$
2. $x \equiv 3 \pmod{5}$, $x \equiv 18 \pmod{20}$
3. $x \equiv 3 \pmod{5}$, $x \equiv 16 \pmod{20}$

True or false: $2^{28} \equiv 1 \pmod{29}$

True or false: $2^{32} \equiv 1 \pmod{33}$

Induction

Define: A *curious* number is a number that is curious. Suppose we know two things about curious numbers:

1. If any integer n is a curious number, then $n + 2$ is a curious number.
2. 7 is a curious number.

Which numbers *must* also be curious?

1. 5
2. 9
3. 10
4. 15
5. 1341
6. 2808
7. 39523092357
8. n
9. ∞

There is a machine that makes widgets all day. It has one problem— if a widget it makes is defective, then the next widget it makes will also be defective. What can you say about the machine?

Recursion

Define $f_n = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ f_{n-1} + f_{n-2} & n \geq 2 \end{cases}$. Find f_2, f_3, f_4 and f_5.

Which Fibonnaci numbers are even? Which ones are divisible by 3?

Because of the Euclidean Algorithm, we can compute the gcd of two non-negative integers with a recursive function: $\gcd(a, b) = \begin{cases} a & b = 0 \\ \gcd(b, a) & b \geq a \\ \gcd(a - b, b) & 0 < b < a \end{cases}$. Use this definition to find $\gcd(13, 8)$.